Worksheet DEQ's #4: Differential Equations Solutions for pages 155 and 157

DEQ's #4 Worksheet Solutions

Tuesday 1/8/18 - Differential Equations

p. 155

25. $f'(x) = 3x - 1$; $f(2) = 3$

$f(x) = \frac{3}{2}x^2 - x + c$

$f(2) = 3$

$3 = \frac{3}{2}(2)^2 - 2 + c$

$c = -1$

$\boxed{f(x) = \frac{3}{2}x^2 - x - 1}$

26. $f'(x) = x^2 - 2x + 2$; $f(3) = -1$

$f'(x) = \frac{1}{3}x^3 - x^2 + 2x + c$

$f(3) = -1$

$-1 = \frac{1}{3}(3)^3 - (3)^2 + 2(3) + c$

$-1 = 9 - 9 + 6 + c$

$c = -7$

$\boxed{f(x) = \frac{1}{3}x^3 - x^2 + 2x - 7}$

27. $f''(x) = 2$; $f'(4) = 6$, $f(4) = 2$

$f''(x) = 2$

$f'(x) = 2x + c$

$f'(4) = 6$

$6 = 2(4) + c$

$c = -2$

$f'(x) = 2x - 2$

$f(x) = x^2 - 2x + c$

$f(4) = 2$

$2 = 4^2 - 2(4) + c$

$c = -6$

$\boxed{f(x) = x^2 - 2x - 6}$

28. $f''(x)=2x;\ f'(-3)=0;\ f(-3)=10$

$$f''(x)=2x$$
$$f'(x)=x^2+C$$
$$f'(-3)=0$$
$$0=(-3)^2+C$$
$$C=-9$$
$$f'(x)=x^2-9$$
$$f(x)=\tfrac{1}{3}x^3-9x+C$$
$$f(-3)=10$$
$$10=\tfrac{1}{3}(-3)^3-9(-3)+C$$
$$10=-9+27+C$$
$$C=-8$$
$$\boxed{f(x)=\tfrac{1}{3}x^3-9x-8}$$

29. $f'(x)=1-\sin x\ ;\ f(0)=\pi$

$$f'(x)=1-\sin x$$
$$f(x)=x-\cos x+C$$
$$f(0)=\pi$$
$$\pi=0-\cos 0+C$$
$$C=\pi-1$$
$$f(x)=x-\cos x+\pi-1$$

therefore,

$$f(\pi)=\pi-\cos\pi+\pi-1=\boxed{2\pi-2}$$

p.157

19. $\int \frac{3\cos x}{7}\,dx = \frac{3}{7}\int \cos x\,dx = \boxed{\frac{3}{7}\sin x + C}$

20. $\int (4-8\cos x)\,dx = \int 4\,dx - \int 8\cos x\,dx = \boxed{4x - 8\sin x + C}$

21. $\int \left(\frac{1}{x^2} - 10\sin x\right)dx = \int x^{-2}\,dx - 10\int \sin x\,dx = \boxed{-\frac{1}{x} + 10\cos x + C}$

22. $\int (\sec^2 t - \csc^2 t + 1)\,dt = \int \sec^2 t\,dt - \int \csc^2 t\,dt + \int dt$

$= \boxed{\tan t + \cot t + t + C}$

23. $\int (\sin^2 x + \cos^2 x)\,dx = \int dx = \boxed{x + C}$

24. $\int \frac{-4\cos x}{1-\cos^2 x}\,dx = -4\int \frac{\cos x}{\sin^2 x}\,dx = -4\int \left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right)dx$

$= 4\int -\csc x \cot x\,dx = \boxed{4\csc x + C}$

25. $f''(x)=2;\ f'(1)=4;\ f(1)=8$

$f''(x) = 2$
$f'(x) = 2x + C$
$f'(1) = 4$
$4 = 2(1) + C$
$C = 2$
$f'(x) = 2x + 2$
$f(x) = x^2 + 2x + C$
$f(1) = 8$
$8 = 1^2 + 2(1) + C$
$C = 5$
$\boxed{f(x) = x^2 + 2x + 5}$

26. $f''(x) = 6x$; $f'(-3) = 20$; $f(-3) = -5$

$f''(x) = 6x$

$f'(x) = 3x^2 + C$

$f'(-3) = 20$, so $20 = 27 + C$

$C = -7$

$f'(x) = 3x^2 - 7$

$f(x) = x^3 - 7x + C$

$f(-3) = -5$, so $-5 = -27 + 21 + C$

$C = 1$

$\boxed{f(x) = x^3 - 7x + 1}$

27. $f''(x) = x^{-\frac{3}{2}}$; $f'(4) = 2$, $f(4) = 0$

$f''(x) = x^{-\frac{3}{2}}$

$f'(x) = -2x^{-\frac{1}{2}} + C$

$f'(4) = 2$, so $2 = -1 + C$

$C = 3$

$f'(x) = -2x^{-\frac{1}{2}} + 3$

$f(x) = -4x^{\frac{1}{2}} + 3x + C$

$f(4) = 0$, so $0 = -8 + 12 + C$

$C = -4$

$\boxed{f(x) = -4x^{\frac{1}{2}} + 3x - 4}$

28. $f''(x) = \cos x$; $f'(\pi) = 2$, $f(\pi) = -1$

$f''(x) = \cos x$

$f'(x) = \sin x + C$

$f'(\pi) = 2$, so $2 = \sin \pi + C$

$C = 2$

$f'(x) = \sin x + 2$

$f(x) = -\cos x + 2x + C$

$f(\pi) = -1$, so $-1 = -\cos \pi + 2\pi + C$

$C = -2 - 2\pi$

$\boxed{f(x) = -\cos x + 2x - 2 - 2\pi}$

29. 100 cats in year 2012 $\longrightarrow$ $f(0) = 100$

Rate of population $\longrightarrow$ $f'(t) = -\frac{1}{4}t - 2$

Find $f(8)$.

$$f'(t) = -\frac{1}{4}t - 2$$

$$f(t) = -\frac{1}{8}t^2 - 2t + C$$

$f(0) = 100$, so $100 = -\frac{1}{8}(0)^2 - 2(0) + C$

$$C = 100$$

$$f(t) = -\frac{1}{8}t^2 - 2t + 100$$

and

$$f(8) = -\frac{1}{8}(64) - 2(8) + 100 = \boxed{76 \text{ cats}}$$