Important Theorems!   Remember these!!!

Extreme value theorem

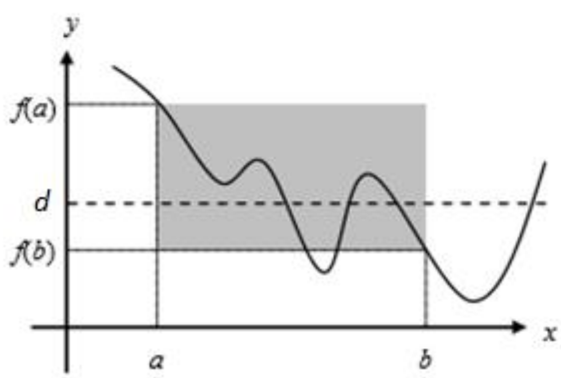
If a real-valued function f is continuous in the closed and bounded interval [a, b], then f must attain a maximum and a minimum, each at least once.

INTERMEDIATE VALUE THEOREM

If f is continuous on the closed interval [a, b] then f takes on every value between $f(a)$ and $f(b)$.

Suppose d is any number between $f(a)$ and $f(b)$, then there is at least one number c in [a, b] such that $f(c) = d$

♪: The Intermediate value theorem tells you that at least one c exists, but it does not give you a method for finding c … you must do that algebraically or graphically.



- Is f continuous on [a, b] ?
- Is $f(b) < d < f(a)$?
- In this example, if $a < c < b$, then there are 3 c's such that $f(c) = d$

Intermediate Value Theorem for Derivatives

If a and b are any two points in an interval on which f is differentiable, then f' takes on every value between $f'(a)$ and $f'(b)$.

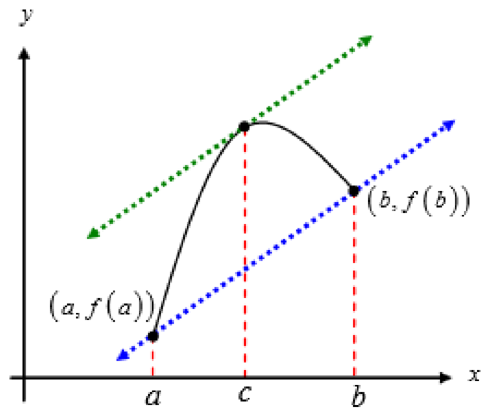
MEAN VALUE THEOREM

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then *there exists* a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Basically, the MVT says …

Instantaneous Rate of Change (slope of a tangent line at some point c)	=	**Average Rate of Change** (slope of secant line joining the endpoints)



Rolle's Theorem

Suppose $f(x)$ is a function that satisfies all of the following.

1. $f(x)$ is continuous on the closed interval $[a,b]$.
2. $f(x)$ is differentiable on the open interval (a,b).
3. $f(a) = f(b)$

Then there is a number c such that $a < c < b$ and $f'(c) = 0$.
Or, in other words $f(x)$ has a critical point in (a,b).