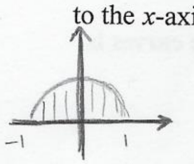
Calculus

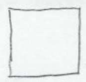
Name______________________________

Review of Volume

For #1 - 7, find the volume of the specified solid. Set up the integral, then evaluate!

1) The base of a solid is the region enclosed by $y = -x^2 + 1$ and $y = 0$. Cross-sections perpendicular to the x-axis are squares.



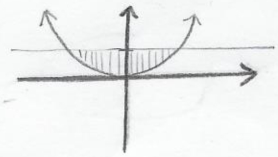


$s = -x^2+1$

$A(x) = (-x^2+1)^2$

$V = \int_{-1}^{1} (-x^2+1)^2\,dx = 1.067$

2) The base of a solid is the region enclosed by $y = 1$ and $y = \frac{x^2}{4}$. Cross-sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.



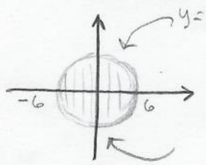
$\text{side} = \frac{1}{\sqrt{2}}\left(1-\frac{x^2}{4}\right)$

$\text{Hyp} = 1-\frac{x^2}{4}$

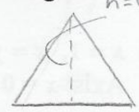
$\text{Area} = \frac{1}{2}s^2 = \frac{1}{2}\left(\frac{1}{\sqrt{2}}\left(1-\frac{x^2}{4}\right)\right)^2$

$V = \int_{-2}^{2} \left(\frac{1}{2}\left(\frac{1}{\sqrt{2}}\left(1-\frac{x^2}{4}\right)\right)^2\right) dx = 0.533$

3) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 36$. Cross-sections perpendicular to the x-axis are equilateral triangles.



$y = \sqrt{36-x^2}$, $y = -\sqrt{36-x^2}$



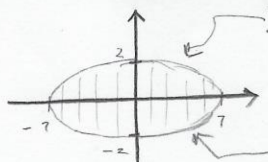
$h = \sqrt{3}\sqrt{36-x^2}$, $2\sqrt{36-x^2}$

$\text{Area} = \frac{1}{2}\left(2\sqrt{36-x^2}\right)\left(\sqrt{3}\sqrt{36-x^2}\right)$

$V = \int_{-6}^{6} \left(\frac{1}{2}\left(2\sqrt{36-x^2}\right)\left(\sqrt{3}\sqrt{36-x^2}\right)\right) dx$

$= 498.831$

4) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{49} + \frac{y^2}{4} = 1$. Cross-sections perpendicular to the x-axis are semicircles.



$y = +\sqrt{4-\frac{4x^2}{9}}$, $y = -\sqrt{4-\frac{4x^2}{9}}$

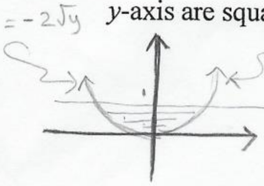
$r = \sqrt{4-\frac{4x^2}{9}}$

$\text{Area} = \frac{1}{2}\pi\left(\sqrt{4-\frac{4x^2}{9}}\right)^2$

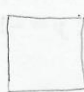
$V = \int_{-7}^{7} \left(\frac{1}{2}\pi\left(\sqrt{4-\frac{4x^2}{9}}\right)^2\right) dx$

$= 58.643$

5) The base of a solid is the region enclosed by $y = 1$ and $y = \frac{x^2}{4}$. Cross-sections perpendicular to the y-axis are squares.



$y = -2\sqrt{y}$, $x = 2\sqrt{y}$

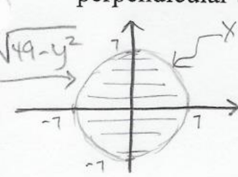


$2\sqrt{y} - (-2\sqrt{y}) = 4\sqrt{y}$

$\text{Area} = (4\sqrt{y})^2$

$V = \int_{0}^{1} (4\sqrt{y})^2\,dy = 8$

6) The base of a solid is the region enclosed by a circle with a diameter of 14. Cross-sections perpendicular to the y-axis are rectangles with heights half that of the side in the xy-plane. $x^2+y^2=7^2$



$x = -\sqrt{49-y^2}$, $x = +\sqrt{49-y^2}$

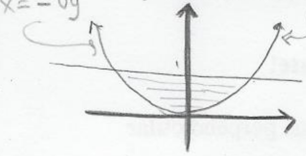
$\sqrt{49-y^2}$, $2\sqrt{49-y^2}$

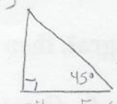
$\text{Area} = \left(2\sqrt{49-y^2}\right)\left(\sqrt{49-y^2}\right)$

$V = \int_{-7}^{7} \left(2\sqrt{49-y^2}\right)\left(\sqrt{49-y^2}\right) dy = 914.667$

7) The base of a solid is the region enclosed by $y = 1$ and $y = x^2$. Cross-sections perpendicular to the y-axis are isosceles right triangles with one leg in the xy-plane.

$x = -\sqrt{y}$ $\quad$ $x = \sqrt{y}$





45°

side $= \sqrt{y} - (-\sqrt{y})$
$= 2\sqrt{y}$

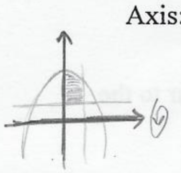
Area $= \frac{1}{2}s^2$
$= \frac{1}{2}(2\sqrt{y})^2$

$V = \int_0^1 \frac{1}{2}(2\sqrt{y})^2\,dy = 1$

For #8 - 19, find the volume of the solid that results when the region enclosed by the curves is revolved about the the given axis. Set up the integral, then evaluate!

8) $y = -x^2 + 6,\ y = 2,\ x = 0,\ x = 2$
Axis: $y = 0$

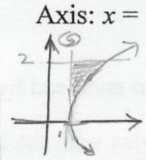
$R = (-x^2+6) - 0$
$r = 2 - 0$



$V = \pi \int_0^2 \left((-x^2+6)^2 - 2^2\right) dx = 120.637$

9) $x = y^2 + 1,\ x = 1,\ y = 2$
Axis: $x = 1$

$R = (y^2+1) - (1)$
$r = 0$



$V = \pi \int_0^2 \left((y^2+1) - 1\right)^2 dy = 20.106$

10) $y = \sqrt{x} + 2,\ y = 2,\ x = 1$
Axis: $y = 0$

11) $x = -y^2 + 1,\ x = 0$
Axis: $x = -2$

#10-19
next page!

12) $y = \sqrt{x + 4},\ y = 1,\ x = -4$
Axis: $x = 1$

13) $x = 1,\ x = y^3,\ y = 0$
Axis: $x = 0$

14) $y = x^2 - 3,\ y = -x^2 - 1$
Axis: $y = 2$

15) $y = x^2 + 1,\ y = 1,\ x = 2$
Axis: $y = 1$

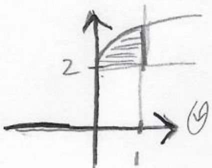
16) $x = -y^2,\ x = -1,\ y = 0,\ y = 1$
Axis: $x = -2$

17) $y = -x^2 + 5,\ y = 1,\ x = 0,\ x = 2$
Axis: $y = -1$

18) $y = x^2 + 1,\ y = 0,\ x = -1,\ x = 2$
Axis: $y = 0$

19) $x = \sqrt{-y},\ x = 0,\ y = -1$
Axis: $y = -1$

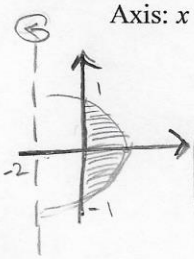
10) $y=\sqrt{x}+2,\ y=2,\ x=1$
Axis: $y=0$ dx Washer



$R=\sqrt{x}+2-0$
$r=2-0$

$$V=\pi\int_0^1\left((\sqrt{x}+2)^2-2^2\right)dx = 9.948\ u^3$$

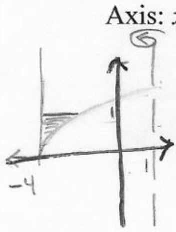
11) $x=-y^2+1,\ x=0$
Axis: $x=-2$ dy Washer



$R=(-y^2+1)-(-2)$
$r=0-(-2)$

$$V=\pi\int_{-1}^1\left((-y^2+3)^2-2^2\right)dy = 20.106\ u^3$$

12) $y=\sqrt{x+4},\ y=1,\ x=-4$
Axis: $x=1$ dy Washer

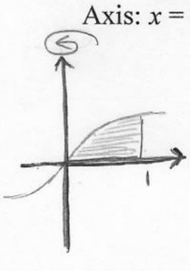


$y=\sqrt{x+4}$
$x=y^2-4$

$R=1-(-4)$
$r=1-(y^2-4)$

$$V=\pi\int_0^1\left(5^2-(5-y^2)^2\right)dy = 9.844\ u^3$$

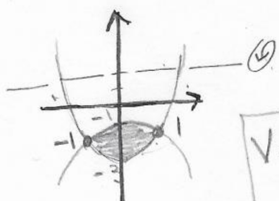
13) $x=1,\ x=y^3,\ y=0$
Axis: $x=0$ dy Washer



$R=1-0$
$r=y^3-0$

$$V=\pi\int_0^1\left(1^2-(y^3)^2\right)dy = 2.693\ u^3$$

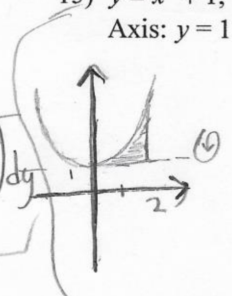
14) $y=x^2-3,\ y=-x^2-1$
Axis: $y=2$ dx Washer



$R=2-(x^2-3)$
$r=2-(-x^2-1)$

$$V=\pi\int_{-1}^1\left((5-x^2)^2-(3+x^2)^2\right)dy = 67.021\ u^3$$

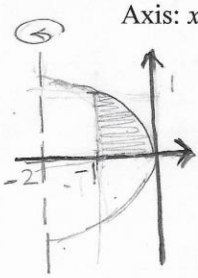
15) $y=x^2+1,\ y=1,\ x=2$
Axis: $y=1$ dx Disk



$r=x^2+1-1$

$$V=\pi\int_0^2(x^2)^2\,dx = 20.106\ u^3$$

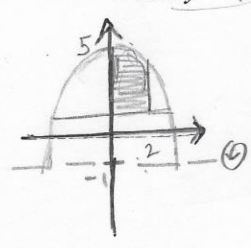
16) $x=-y^2,\ x=-1,\ y=0,\ y=1$
Axis: $x=-2$ dy Washer



$R=-y^2-(-2)$
$r=-1-(-2)$

$$V=\pi\int_0^1\left((-y^2+2)^2-1^2\right)dy = 5.864\ u^3$$

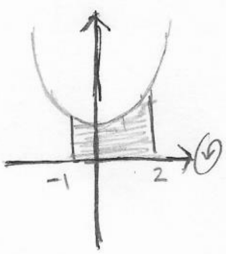
17) $y=-x^2+5,\ y=1,\ x=0,\ x=2$
Axis: $y=-1$ dx Washer



$R=-x^2+5-(-1)$
$r=1-(-1)$

$$V=\pi\int_0^2\left((-x^2+6)^2-(2^2)\right)dx = 120.637\ u^3$$

18) $y=x^2+1,\ y=0,\ x=-1,\ x=2$
Axis: $y=0$ dx Disk

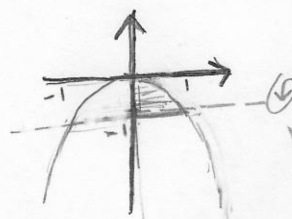


$r=x^2+1-0$

$$V=\pi\int_{-1}^2(x^2+1)^2\,dx = 49.009\ u^3$$

19) $x=\sqrt{-y},\ x=0,\ y=-1$
Axis: $y=-1$ dx Disk

$x=\sqrt{-y}$
$y=-x^2$



$r=-x^2-(-1)$

$$V=\pi\int_0^1(-x^2+1)^2\,dx = 1.676\ u^3$$