Some Review for Spring Benchmark 1

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

1) $y = (2x + 4)^{\frac{2}{3}}$ at $(2, 4)$

2) $y = \cos(x)$ at $\left(-\frac{\pi}{2}, 0\right)$

3) $y = \tan(x)$ at $(\pi, 0)$

4) $y = e^{-x - 1}$ at $(-2, e)$

5) $y = \ln(x + 2)$ at $(0, \ln 2)$

For each problem, find the slope of the function at the given value.

6) $y = (3x + 6)^{\frac{1}{2}}$ at $x = 3$

7) $y = 2\sec(x)$ at $x = \frac{\pi}{4}$

8) $y = \ln(-x + 2)$ at $x = 0$

9) $y = e^{x + 1}$ at $x = 0$

Solve each related rate problem.

10) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 2 cm/min. How fast is the area of the pool increasing when the radius is 14 cm?

11) A spherical snowball is rolled in fresh snow, causing it to grow so that its radius increases at a rate of 2 in/sec. How fast is the volume of the snowball increasing when the radius is 4 in?

12) A conical paper cup is 20 cm tall with a radius of 10 cm. The cup is being filled with water so that the water level rises at a rate of 3 cm/sec. At what rate is water being poured into the cup when the water level is 7 cm?

For each problem, find the values of *c* that satisfy the Mean Value Theorem.

13) $y = \frac{x^2}{2} + 4x + 7;\ \ [-7, -3]$

14) $y = 2x^2 + 8x + 10;\ \ [-3, -1]$

For each problem, find the open intervals where the function is increasing and decreasing.

15) $y = -x^3 - 14x^2 - 60x - 78$

16) $y = x^3 - 5x^2 + 7x - 2$

For each problem, find the open intervals where the function is concave up and concave down.

17) $y = -x^3 + 4x^2 - 7$

18) $y = -x^4 + x^2 - 1$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$, the acceleration function $a(t)$, the times t when the particle changes directions, the intervals of time when the particle is moving left and moving right, and the intervals of time when the particle is slowing down and speeding up.

19) $s(t) = t^3 - 20t^2 + 100t$

20) $s(t) = t^3 - 9t^2$

For each problem, use a right-hand Riemann sum to approximate the integral based off of the values in the table.

21) $\int_0^9 f(x)\,dx$

x	0	2	3	7	9
$f(x)$	4	5	6	4	3

22) $\int_0^9 f(x)\,dx$

x	0	4	6	7	9
$f(x)$	5	7	6	4	5

Differentiate each function with respect to *x*.

23) $y = 2x^3(2x^3 - 2)$

24) $y = -x^2(4x^2 + 5)$

25) $y = \dfrac{4x^2}{5x^4 + 5}$

26) $y = \dfrac{x^5 + 4x^4}{4x^4 - 4}$

27) $y = (x^5 + 3)^5$

28) $y = (3x + 1)^{\frac{1}{5}}$

29) $y = e^{5x^5}$

30) $y = \ln 2x^3$

31) $y = \ln 5x^5$

32) $y = \ln 5x^3$

33) $y = \csc 2x^3$

34) $y = \sin 2x^2$

35) $y = \sec 3x^5$

36) $y = \tan 5x^5$

37) $y = \csc 3x^3$

38) $y = \sec x^5$

For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

39)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-2	2	2
2	1	$-\frac{1}{2}$	4	0
3	2	1	2	$-\frac{3}{2}$
4	3	1	1	-1

Part 1) Given $h_1(x) = f(x) + g(x)$, find $h_1'(4)$

Part 2) Given $h_2(x) = f(x) - g(x)$, find $h_2'(1)$

Part 3) Given $h_3(x) = f(x) \cdot g(x)$, find $h_3'(1)$

Part 4) Given $h_4(x) = \dfrac{f(x)}{g(x)}$, find $h_4'(3)$

Part 5) Given $h_5(x) = (f(x))^2$, find $h_5'(2)$

Part 6) Given $h_6(x) = f(g(x))$, find $h_6'(2)$

Evaluate each indefinite integral.

40) $\int 1\, dx$

41) $\int \frac{15x^{\frac{1}{2}}}{2}\, dx$

42) $\int 8x^3(2x^4 - 5)^3\, dx$

43) $\int 5\csc^2 -5x \cdot \cot^3 -5x\, dx$

44) $\int \frac{(5 + \ln -3x)^3}{x}\, dx$

45) $\int 3e^{3x} \cdot (e^{3x} - 4)^4\, dx$

A particle moves along a coordinate line. Its velocity function is $v(t)$ for $t \geq 0$. For each problem, find the displacement of the particle and the distance traveled by the particle over the given interval.

46) $v(t) = -2t + 26;\ \ 7 \leq t \leq 16$

Solve each optimization problem.

47) A supermarket employee wants to construct an open-top box from a 14 by 30 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

Evaluate each definite integral.

48) $\int_{-2}^{3}\left(-\frac{x^2}{2}+x-\frac{1}{2}\right)dx$

49) $\int_{-1}^{0}(x^2+2)\,dx$

50) $\int_{-1}^{1}2e^x\,dx$

51) $\int_{-2}^{1}-2e^x\,dx$

52) $\int_{-3}^{-2}\frac{3}{x}\,dx$

53) $\int_{2}^{5}\frac{5}{x}\,dx$

54) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{6}}-2\sin x\,dx$

55) $\int_{0}^{\frac{\pi}{4}}\sec^2 x\,dx$

56) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}2\csc^2 x\,dx$

57) $\int_{-\frac{\pi}{4}}^{0}-\sec x\tan x\,dx$

58) $\int_{-5}^{-3}-e^{x+3}\,dx$

59) $\int_{-4}^{-2}\frac{1}{(x+1)^2}\,dx$

Answers to Some Review for Spring Benchmark 1

1) $y = \frac{2}{3}x + \frac{8}{3}$
2) $y = x + \frac{\pi}{2}$
3) $y = x - \pi$
4) $y = -ex - e$
5) $y = \frac{1}{2}x + \ln 2$
6) $\frac{\sqrt{15}}{10}$
7) $2\sqrt{2}$
8) $-\frac{1}{2}$
9) e

10) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dr}{dt} = 2$ Find: $\left.\frac{dA}{dt}\right|_{r=14}$

$\left.\frac{dA}{dt}\right|_{r=14} = 2\pi r \cdot \frac{dr}{dt} = 56\pi$ cm²/min

11) V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dr}{dt} = 2$ Find: $\left.\frac{dV}{dt}\right|_{r=4}$

$\left.\frac{dV}{dt}\right|_{r=4} = 4\pi r^2 \cdot \frac{dr}{dt} = 128\pi$ in³/sec

12) V = volume of material in cone h = height t = time

Equation: $V = \frac{\pi h^3}{12}$ Given rate: $\frac{dh}{dt} = 3$ Find: $\left.\frac{dV}{dt}\right|_{h=7}$

$\left.\frac{dV}{dt}\right|_{h=7} = \frac{\pi h^2}{4} \cdot \frac{dh}{dt} = \frac{147\pi}{4}$ cm³/sec

13) $\{-5\}$
14) $\{-2\}$

15) Increasing: $\left(-6, -\frac{10}{3}\right)$ Decreasing: $(-\infty, -6), \left(-\frac{10}{3}, \infty\right)$

16) Increasing: $(-\infty, 1), \left(\frac{7}{3}, \infty\right)$ Decreasing: $\left(1, \frac{7}{3}\right)$

17) Concave up: $\left(-\infty, \frac{4}{3}\right)$ Concave down: $\left(\frac{4}{3}, \infty\right)$

18) Concave up: $\left(-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{6}}{6}\right), \left(\frac{\sqrt{6}}{6}, \infty\right)$

19) $v(t) = 3t^2 - 40t + 100$, $a(t) = 6t - 40$

Changes direction at: $t = \left\{\frac{10}{3}, 10\right\}$, Moving left: $\frac{10}{3} < t < 10$, Moving right: $0 \le t < \frac{10}{3}$, $t > 10$

Slowing down: $0 \le t < \frac{10}{3}$, $\frac{20}{3} < t < 10$, Speeding up: $\frac{10}{3} < t < \frac{20}{3}$, $t > 10$

20) $v(t) = 3t^2 - 18t$, $a(t) = 6t - 18$

Changes direction at: $t = \{6\}$, Moving left: $0 < t < 6$, Moving right: $t > 6$

Slowing down: $3 < t < 6$, Speeding up: $0 < t < 3$, $t > 6$

21) 38
22) 54
23) $\frac{dy}{dx} = 2x^3 \cdot 6x^2 + (2x^3 - 2) \cdot 6x^2$
$= 24x^5 - 12x^2$

24) $\frac{dy}{dx} = -x^2 \cdot 8x + (4x^2 + 5) \cdot -2x$
$= -16x^3 - 10x$

25) $\frac{dy}{dx} = \frac{(5x^4 + 5) \cdot 8x - 4x^2 \cdot 20x^3}{(5x^4 + 5)^2}$
$= \frac{-8x^5 + 8x}{5x^8 + 10x^4 + 5}$

26) $\frac{dy}{dx} = \frac{(4x^4 - 4)(5x^4 + 16x^3) - (x^5 + 4x^4) \cdot 16x^3}{(4x^4 - 4)^2}$
$= \frac{x^8 - 5x^4 - 16x^3}{4x^8 - 8x^4 + 4}$

27) $\frac{dy}{dx} = 5(x^5 + 3)^4 \cdot 5x^4$
$= 25x^4(x^5 + 3)^4$

28) $\frac{dy}{dx} = \frac{1}{5}(3x + 1)^{-\frac{4}{5}} \cdot 3$
$= \frac{3}{5(3x + 1)^{\frac{4}{5}}}$

29) $\frac{dy}{dx} = e^{5x^5} \cdot 25x^4$

30) $\frac{dy}{dx} = \frac{1}{2x^3} \cdot 6x^2$
$= \frac{3}{x}$

31) $\frac{dy}{dx} = \frac{1}{5x^5} \cdot 25x^4$
$= \frac{5}{x}$

32) $\frac{dy}{dx} = \frac{1}{5x^3} \cdot 15x^2$
$= \frac{3}{x}$

33) $\frac{dy}{dx} = -\csc 2x^3 \cot 2x^3 \cdot 6x^2$
$= -6x^2 \csc 2x^3 \cot 2x^3$

34) $\frac{dy}{dx} = \cos 2x^2 \cdot 4x$
$= 4x \cos 2x^2$

35) $\frac{dy}{dx} = \sec 3x^5 \tan 3x^5 \cdot 15x^4$
$= 15x^4 \sec 3x^5 \tan 3x^5$

36) $\frac{dy}{dx} = \sec^2 5x^5 \cdot 25x^4$
$= 25x^4 \sec^2 5x^5$

37) $\frac{dy}{dx} = -\csc 3x^3 \cot 3x^3 \cdot 9x^2$
$= -9x^2 \csc 3x^3 \cot 3x^3$

38) $\frac{dy}{dx} = \sec x^5 \tan x^5 \cdot 5x^4$
$= 5x^4 \sec x^5 \tan x^5$

39) $h_1'(4) = 0$
$h_2'(1) = -4$
$h_3'(1) = 2$
$h_4'(3) = \frac{5}{4}$
$h_5'(2) = -1$
$h_6'(2) = 0$

40) $x + C$

41) $5x^{\frac{3}{2}} + C$

42) $\frac{1}{4}(2x^4 - 5)^4 + C$

43) $\frac{1}{4} \cdot \cot^4 -5x + C$

44) $\frac{1}{4}(5 + \ln -3x)^4 + C$

45) $\frac{1}{5}(e^{3x} - 4)^5 + C$

46) Displacement: 27
Distance traveled: 45

47) 3 in

48) $-\frac{35}{6} \approx -5.833$

49) $\frac{7}{3} \approx 2.333$

50) $\frac{2e^2 - 2}{e} \approx 4.701$

51) $\frac{-2e^3 + 2}{e^2} \approx -5.166$

52) $3 \ln 2 - 3 \ln 3 \approx -1.216$

53) $5 \ln 5 - 5 \ln 2 \approx 4.581$

54) $\sqrt{3} - \sqrt{2} \approx 0.318$

55) 1

56) $\frac{2\sqrt{3}}{3} \approx 1.155$

57) $-1 + \sqrt{2} \approx 0.414$

58) $\frac{-e^2 + 1}{e^2} \approx -0.865$

59) $\frac{2}{3} \approx 0.667$

