Calculus Name_____

More review for Benchmark 1

Differentiate each function with respect to *x*.

1) $y = 2x^2(2x^3 + 3)$

2) $y = (x^2 + 1)(3x^5 + 5)$

3) $y = (\sqrt[5]{x^2} + 1)(5x^4 - 5)$

4) $y = \csc 4x^2$

5) $y = \sec 4x^3$

6) $y = \cot x^4$

7) $y = \sin 2x^3$

8) $y = \csc 3x^5$

For each problem, find the open intervals where the function is increasing and decreasing.

9) $y = x^3 - 3x^2 - 2$

10) $y = -x^3 - 8x^2 - 16x - 3$

For each problem, find the open intervals where the function is concave up and concave down.

11) $y = -x^4 + 3x^2 - 4$

12) $y = x^3 - x^2 + 2$

For each problem, find all points of absolute minima and maxima on the given interval.

13) $y = x^2 + 8x + 15;\ \ [-6, -3]$

14) $y = x^3 - 2x^2;\ \ [-1, 1]$

For each problem, find the slope of the function at the given value.

15) $y = 2\cot(x)$ at $x = -\frac{3\pi}{4}$

16) $y = \sec(x)$ at $x = -\frac{\pi}{4}$

17) $y = \sin(x)$ at $x = -\pi$

18) $y = 2\sin(2x)$ at $x = \frac{3\pi}{4}$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

19) $x^2 + 4y^3 = 5$

20) $5y^2 = 4x^2 - 5x^2y$

21) $1 = 4x^2 - 3x^3y$

22) $5x^3y^3 = x^3 + 3x^3y^2$

Evaluate each limit.

23) $\lim\limits_{x \to 3^+} -\frac{2}{x^2 - 9}$

24) $\lim\limits_{x \to 2^-} \frac{x}{x - 2}$

25) $\lim\limits_{x \to -1} -\frac{x^2}{x + 1}$

26) $\lim\limits_{x \to -1} \frac{x + 2}{x^2 + 3x + 2}$

27) $\lim_{x \to \infty} \frac{x}{2x^2 + 2x + 1}$

28) $\lim_{x \to \infty} \frac{x^4}{x^2 + 2}$

29) $\lim_{x \to 1^+} f(x),\ f(x) = \begin{cases} x + 2, & x \le 1 \\ -\frac{x}{2} - 1, & x > 1 \end{cases}$

30) $\lim_{x \to -2^-} f(x),\ f(x) = \begin{cases} -x - 1, & x \le -2 \\ 2x + 2, & x > -2 \end{cases}$

31) $\lim_{x \to 3} \frac{x^2 - 4x + 3}{x - 3}$

32) $\lim_{x \to -3} -\frac{x^2 - 9}{x + 3}$

33) $\lim_{x \to -3} \left(x^2 + 8x + 15\right)$

34) $\lim_{x \to -1} \frac{x^2 + x - 6}{x + 3}$

Evaluate each limit. Use L'Hôpital's Rule if it can be applied. If it cannot be applied, evaluate using another method and write a * next to your answer.

35) $\lim_{x \to 1} \frac{3(x - 1)}{\ln x}$

36) $\lim_{x \to 0} \frac{2\left(e^x - e^{-x}\right)}{x}$

37) $\lim_{x \to 0} \frac{e^{5x} - 1}{x}$

38) $\lim_{x \to 0^+} \frac{2\left(e^x + e^{-x}\right)}{x}$

Evaluate each indefinite integral.

39) $\int \left(5x^5 - 3\right)^4 \cdot 100x^4 \, dx$

40) $\int \left(5x^4 - 4\right)^3 \cdot 40x^3 \, dx$

41) $\int 32x^3(4x^4 - 5)^4\,dx$

42) $\int (5x^4 + 1)^5 \cdot 80x^3\,dx$

For each problem, find all points of relative minima and maxima.

43) $y = -x^3 + 3x^2 - 6$

44) $y = -x^3 + 3x^2 + 1$

For each problem, approximate the area under the curve over the given interval using 4 left endpoint rectangles.

45) $y = x + 4;\ [-2, 6]$

46) $y = -x^2 + 12;\ [-3, 1]$

For each problem, approximate the area under the curve over the given interval using 4 right endpoint rectangles.

47) $y = -x^2 + 13;\ [-2, 2]$

48) $y = \frac{x^2}{2} + x + 2;\ [-5, 3]$

For each problem, approximate the area under the curve over the given interval using 4 midpoint rectangles.

49) $y = -x + 6;\ [-2, 2]$

50) $y = \frac{x^2}{2} - x + 2;\ [1, 5]$

For each problem, approximate the area under the curve over the given interval using 4 trapezoids.

51) $y = -x + 4;\ [-1, 3]$

52) $y = x^2 + 2x + 3;\ [-4, 0]$

For each problem, find all points of absolute minima and maxima on the given interval.

53) $y = x^3 - 2x^2 + 2;\ \ [-1, 2]$

54) $y = x^3 - 3x^2 - 2;\ \ [-1, 1]$

Answers to More review for Benchmark 1

1) $\frac{dy}{dx} = 2x^2 \cdot 6x^2 + (2x^3 + 3) \cdot 4x$
$= 20x^4 + 12x$

2) $\frac{dy}{dx} = (x^2 + 1) \cdot 15x^4 + (3x^5 + 5) \cdot 2x$
$= 21x^6 + 15x^4 + 10x$

3) $\frac{dy}{dx} = \left(x^{\frac{2}{5}} + 1\right) \cdot 20x^3 + (5x^4 - 5) \cdot \frac{2}{5}x^{-\frac{3}{5}}$
$= 22x^{\frac{17}{5}} + 20x^3 - \frac{2}{x^{\frac{3}{5}}}$

4) $\frac{dy}{dx} = -\csc 4x^2 \cot 4x^2 \cdot 8x$
$= -8x \csc 4x^2 \cot 4x^2$

5) $\frac{dy}{dx} = \sec 4x^3 \tan 4x^3 \cdot 12x^2$
$= 12x^2 \sec 4x^3 \tan 4x^3$

6) $\frac{dy}{dx} = -\csc^2 x^4 \cdot 4x^3$
$= -4x^3 \csc^2 x^4$

7) $\frac{dy}{dx} = \cos 2x^3 \cdot 6x^2$
$= 6x^2 \cos 2x^3$

8) $\frac{dy}{dx} = -\csc 3x^5 \cot 3x^5 \cdot 15x^4$
$= -15x^4 \csc 3x^5 \cot 3x^5$

9) Increasing: $(-\infty, 0), (2, \infty)$ Decreasing: $(0, 2)$

10) Increasing: $\left(-4, -\frac{4}{3}\right)$ Decreasing: $(-\infty, -4), \left(-\frac{4}{3}, \infty\right)$

11) Concave up: $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{2}}{2}\right), \left(\frac{\sqrt{2}}{2}, \infty\right)$

12) Concave up: $\left(\frac{1}{3}, \infty\right)$ Concave down: $\left(-\infty, \frac{1}{3}\right)$

13) Absolute minimum: $(-4, -1)$
Absolute maximum: $(-6, 3)$

14) Absolute minimum: $(-1, -3)$
Absolute maximum: $(0, 0)$

15) -4

16) $-\sqrt{2}$

17) -1

18) 0

19) $\frac{dy}{dx} = -\frac{x}{6y^2}$

20) $\frac{dy}{dx} = \frac{8x - 10xy}{10y + 5x^2}$

21) $\frac{dy}{dx} = \frac{8 - 9xy}{3x^2}$

22) $\frac{dy}{dx} = \frac{1 + 3y^2 - 5y^3}{5xy^2 - 2xy}$

23) $-\infty$

24) $-\infty$

25) Does not exist.

26) Does not exist.

27) 0

28) ∞

29) $-\frac{3}{2}$

30) 1

31) 2

32) 6

33) 0

34) -3

35) 3

36) 4

37) 5

38) ∞ *

39) $\frac{4}{5}(5x^5 - 3)^5 + C$

40) $\frac{1}{2}(5x^4 - 4)^4 + C$

41) $\frac{2}{5}(4x^4 - 5)^5 + C$

42) $\frac{2}{3}(5x^4 + 1)^6 + C$

43) Relative minimum: $(0, -6)$
Relative maximum: $(2, -2)$

44) Relative minimum: $(0, 1)$
Relative maximum: $(2, 5)$

45) 40

46) 34

47) 46

48) 36

49) 24

50) $\frac{33}{2} = 16.5$

51) 12

52) 18

53) Absolute minimum: $(-1, -1)$
Absolute maxima: $(2, 2), (0, 2)$

54) Absolute minimum: $(-1, -6)$
Absolute maximum: $(0, -2)$