Calculus Name_____

Some Review for Fall Final

For each problem, find the values of *c* that satisfy the Mean Value Theorem.

1) $y = \dfrac{x^2 - 9}{3x}$; $[-4, -1]$

2) $y = -x^3 - 11x^2 - 35x - 31$; $[-5, -3]$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$, the acceleration function $a(t)$, the times *t* when the particle changes directions, the intervals of time when the particle is moving left and moving right, and the intervals of time when the particle is slowing down and speeding up.

3) $s(t) = -t^3 + 10t^2$

For each problem, find all points of relative minima and maxima.

4) $y = -x^3 + 2x^2 - 2$

5) $y = -x^3 - 3x^2 + 6$

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

6) $y = -\sin(2x)$ at $(-\pi, 0)$

7) $y = -e^{x-1}$ at $\left(0, -\frac{1}{e}\right)$

For each problem, find the open intervals where the function is concave up and concave down.

8) $y = -\cos(2x)$; $[-\pi, \pi]$

Evaluate each limit using L'Hôpital's Rule.

9) $\lim\limits_{x \to 0} \frac{3\left(e^{x} - e^{-x}\right)}{\sin x}$

10) $\lim\limits_{x \to \infty} \frac{\ln x^{4}}{\ln (x+1)^{5}}$

Solve each related rate problem.

11) A conical paper cup is 30 cm tall with a radius of 10 cm. The cup is being filled with water at a rate of $\frac{2\pi}{3}$ cm³/sec. How fast is the water level rising when the water level is 4 cm?

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the maximum speed and times t when this speed occurs, the displacement of the particle, and the distance traveled by the particle over the given interval.

12) $s(t) = t^2 - 9t - 10;\ 3 \leq t \leq 5$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

13) $-y + 1 = 3x^2 + 4y^2$

14) $5x^3 + 2x^2y^3 = -5y^2 + 4$

Evaluate each limit.

15) $\lim\limits_{x \to -\infty} -\dfrac{2x^2}{x^2 + 9}$

16) $\lim\limits_{x \to -\infty} \dfrac{2x^2}{3x - 2}$

17) $\lim\limits_{x \to -\infty} -e^{-3x}$

18) $\lim\limits_{x \to \infty} \dfrac{-x + 3}{x^2 + x + 1}$

For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.

19) $y = 2\cos(2x)$ at $\left(-\dfrac{3\pi}{4}, 0\right)$

20) $y = -2\cot(x)$ at $\left(\dfrac{\pi}{2}, 0\right)$

For each problem, find the average rate of change of the function over the given interval.

21) $y = 2x^2 - x + 1; \quad [0, 2]$

22) $y = \cos 2x; \quad [0, \frac{\pi}{2}]$

For each problem, find the open intervals where the function is A) concave up and concave down and B) both decreasing and concave up?

23) $y = 2x^3 - 3x^2 - 12x + 18$

24) $y = x^3 - 4x^2 + 4$

Differentiate each function with respect to *x*.

25) $y = \csc x^2$

26) $y = \cot 3x^3$

27) $y = \sec 2x^5$

28) $y = \tan x^2$

29) $y = \cos 3x^4$

30) $y = \sin 5x^2$

31) $y = \tan\left(2x^2 + 3\right)^2$

32) $y = \tan^3 3x^4$

33) $y = \sin^2 5x^4$

34) $y = \cos^2\left(x^3 + x^2\right)$

35) $y = e^{x^2}$

36) $y = \ln\left(2x^4 - 4\right)$

37) $y = e^{4x^2}$

38) $y = \left(4x^2 + 1\right) \cdot e^{5x^3}$

39) $y = \ln x^5$

40) $y = e^{4x^5}$

Calculus Name_____

Some Review for Fall Final

For each problem, find the values of *c* that satisfy the Mean Value Theorem.

1) $y = \frac{x^2 - 9}{3x}$; $[-4, -1]$

$\{-2\}$

2) $y = -x^3 - 11x^2 - 35x - 31$; $[-5, -3]$

$\left\{-\frac{13}{3}\right\}$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$, the acceleration function $a(t)$, the times t when the particle changes directions, the intervals of time when the particle is moving left and moving right, and the intervals of time when the particle is slowing down and speeding up.

3) $s(t) = -t^3 + 10t^2$

$v(t) = -3t^2 + 20t$, $a(t) = -6t + 20$

Changes direction at: $t = \left\{\frac{20}{3}\right\}$, Moving left: $t > \frac{20}{3}$, Moving right: $0 < t < \frac{20}{3}$

Slowing down: $\frac{10}{3} < t < \frac{20}{3}$, Speeding up: $0 < t < \frac{10}{3}$, $t > \frac{20}{3}$

For each problem, find all points of relative minima and maxima.

4) $y = -x^3 + 2x^2 - 2$

Relative minimum: $(0, -2)$

Relative maximum: $\left(\frac{4}{3}, -\frac{22}{27}\right)$

5) $y = -x^3 - 3x^2 + 6$

Relative minimum: $(-2, 2)$

Relative maximum: $(0, 6)$

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

6) $y = -\sin(2x)$ at $(-\pi, 0)$

$y = -2x - 2\pi$

7) $y = -e^{x-1}$ at $\left(0, -\frac{1}{e}\right)$

$y = -\frac{1}{e} \cdot x - \frac{1}{e}$

For each problem, find the open intervals where the function is concave up and concave down.

8) $y = -\cos(2x)$; $[-\pi, \pi]$

Concave up: $\left(-\pi, -\frac{3\pi}{4}\right), \left(-\frac{\pi}{4}, \frac{\pi}{4}\right), \left(\frac{3\pi}{4}, \pi\right)$ Concave down: $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

Evaluate each limit using L'Hôpital's Rule.

9) $\lim_{x \to 0} \frac{3\left(e^{x} - e^{-x}\right)}{\sin x}$

6

10) $\lim_{x \to \infty} \frac{\ln x^{4}}{\ln (x+1)^{5}}$

$\frac{4}{5}$

Solve each related rate problem.

11) A conical paper cup is 30 cm tall with a radius of 10 cm. The cup is being filled with water at a rate of $\frac{2\pi}{3}$ cm³/sec. How fast is the water level rising when the water level is 4 cm?

$\frac{3}{8}$ cm/sec

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the maximum speed and times t when this speed occurs, the displacement of the particle, and the distance traveled by the particle over the given interval.

12) $s(t) = t^2 - 9t - 10;\ 3 \leq t \leq 5$

Maximum speed: 3 at $t = \{3\}$
Displacement: -2
Distance traveled: $\frac{5}{2} = 2.5$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

13) $-y + 1 = 3x^2 + 4y^2$

$\frac{dy}{dx} = \frac{6x}{-1 - 8y}$

14) $5x^3 + 2x^2y^3 = -5y^2 + 4$

$\frac{dy}{dx} = \frac{-15x^2 - 4xy^3}{6y^2x^2 + 10y}$

Evaluate each limit.

15) $\lim_{x \to -\infty} -\frac{2x^2}{x^2 + 9}$

-2

16) $\lim_{x \to -\infty} \frac{2x^2}{3x - 2}$

$-\infty$

17) $\lim_{x \to -\infty} -e^{-3x}$

$-\infty$

18) $\lim_{x \to \infty} \frac{-x + 3}{x^2 + x + 1}$ 0

For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.

19) $y = 2\cos(2x)$ at $\left(-\frac{3\pi}{4}, 0\right)$

$y = \frac{1}{4}x + \frac{3\pi}{16}$

20) $y = -2\cot(x)$ at $\left(\frac{\pi}{2}, 0\right)$

$y = -\frac{1}{2}x + \frac{\pi}{4}$

For each problem, find the average rate of change of the function over the given interval.

21) $y = 2x^2 - x + 1; \quad [0, 2]$

3

22) $y = \cos 2x; \quad [0, \frac{\pi}{2}]$

$-\frac{4}{\pi}$

For each problem, find the open intervals where the function is A) concave up and concave down and B) both decreasing and concave up?

23) $y = 2x^3 - 3x^2 - 12x + 18$

Concave up: $\left(\frac{1}{2}, \infty\right)$ Concave down: $\left(-\infty, \frac{1}{2}\right)$ B) $\left(\frac{1}{2}, 2\right)$

24) $y = x^3 - 4x^2 + 4$

A) Concave up: $\left(\frac{4}{3}, \infty\right)$ Concave down: $\left(-\infty, \frac{4}{3}\right)$ B) $\left(\frac{4}{3}, \frac{8}{3}\right)$

Differentiate each function with respect to *x*.

25) $y = \csc x^2$

$\frac{dy}{dx} = -\csc x^2 \cot x^2 \cdot 2x$

$= -2x \csc x^2 \cot x^2$

26) $y = \cot 3x^3$

$\frac{dy}{dx} = -\csc^2 3x^3 \cdot 9x^2$

$= -9x^2 \csc^2 3x^3$

27) $y = \sec 2x^5$

$\frac{dy}{dx} = \sec 2x^5 \tan 2x^5 \cdot 10x^4$

$= 10x^4 \sec 2x^5 \tan 2x^5$

28) $y = \tan x^2$

$\frac{dy}{dx} = \sec^2 x^2 \cdot 2x$

$= 2x \sec^2 x^2$

29) $y = \cos 3x^4$

$$\frac{dy}{dx} = -\sin 3x^4 \cdot 12x^3$$
$$= -12x^3 \sin 3x^4$$

30) $y = \sin 5x^2$

$$\frac{dy}{dx} = \cos 5x^2 \cdot 10x$$
$$= 10x\cos 5x^2$$

31) $y = \tan (2x^2 + 3)^2$

$$\frac{dy}{dx} = \sec^2 (2x^2 + 3)^2 \cdot 2(2x^2 + 3) \cdot 4x$$
$$= 8x\sec^2 (2x^2 + 3)^2(2x^2 + 3)$$

32) $y = \tan^3 3x^4$

$$\frac{dy}{dx} = 3\tan^2 3x^4 \cdot \sec^2 3x^4 \cdot 12x^3$$
$$= 36x^3 \tan^2 3x^4 \sec^2 3x^4$$

33) $y = \sin^2 5x^4$

$$\frac{dy}{dx} = 2\sin 5x^4 \cdot \cos 5x^4 \cdot 20x^3$$
$$= 40x^3 \sin 5x^4 \cos 5x^4$$

34) $y = \cos^2 (x^3 + x^2)$

$$\frac{dy}{dx} = -2(3x^2 + 2x)\cos (x^3 + x^2)\sin (x^3 + x^2)$$

35) $y = e^{x^2}$

$$\frac{dy}{dx} = 2xe^{x^2}$$

36) $y = \ln (2x^4 - 4)$

$$\frac{dy}{dx} = \frac{4x^3}{x^4 - 2}$$

37) $y = e^{4x^2}$

$$\frac{dy}{dx} = 8xe^{4x^2}$$

38) $y = (4x^2 + 1) \cdot e^{5x^3}$

$$\frac{dy}{dx} = xe^{5x^3}(60x^3 + 15x + 8)$$

39) $y = \ln x^5$

$$\frac{dy}{dx} = \frac{5}{x}$$

40) $y = e^{4x^5}$

$$\frac{dy}{dx} = 20x^4 e^{4x^5}$$