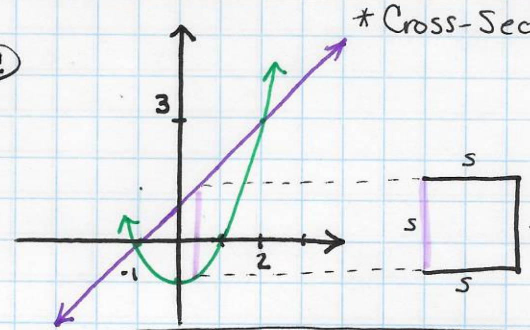
Answers to Worksheet on Volume by Cross Sections

#1

ⓐ



* Cross-Sections are perpendicular to x-axis.

Cross-section is a square:

side = top - bottom

$s = (x+1) - (x^2 - 1)$

$A(x) = s^2 = \left((x+1) - (x^2-1)\right)^2$

$$\boxed{V = \int_a^b A(x)\,dx = \int_{-1}^{2} \left((x+1) - (x^2-1)\right)^2 dx = 8.1\ u^3}$$

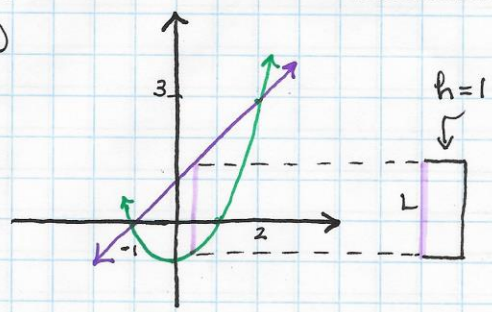
* Remember: Find interval by setting the two functions equal and solving for x.

$x+1 = x^2 - 1$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$ so $x = -1, 2$

ⓑ



Cross-section is a rectangle of height 1:

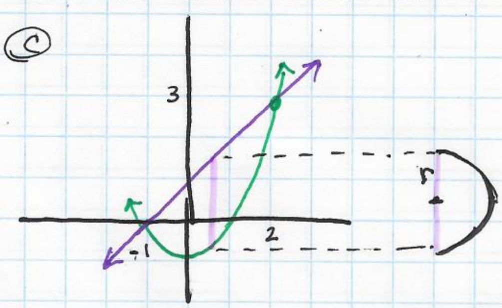
Length = top − bottom

$L = (x+1) - (x^2-1)$

$A(x) = Lh = \left((x+1) - (x^2-1)\right)(1)$

$$\boxed{V = \int_a^b A(x)\,dx = \int_{-1}^{2} \left((x+1) - (x^2-1)\right)(1)\,dx = 4.5\ u^3}$$

ⓒ



Cross-sections are semi-circles.

radius = $\frac{1}{2}$ diameter = $\frac{1}{2}$(top − bottom)

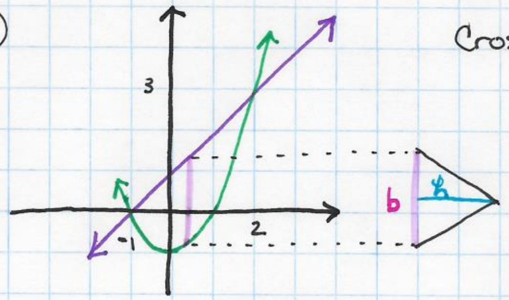
$r = \frac{1}{2}\left((x+1) - (x^2-1)\right)$

$A(x) = \frac{1}{2}(\pi r^2) = \frac{1}{2}\pi\left(\frac{1}{2}\left((x+1) - (x^2-1)\right)\right)^2$

$$\boxed{V = \int_a^b A(x)\,dx = \int_{-1}^{2} \frac{1}{2}\pi\left(\frac{1}{2}\left((x+1) - (x^2-1)\right)\right)^2 dx = 3.181\ u^3}$$

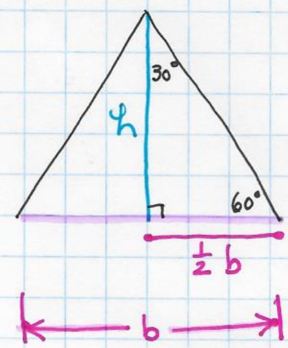
Answers to Worksheet on Volume by Cross Sections

(d)



Cross-section is an Equilateral Triangle:

Turn the triangle:



special triangle:

$h = \left(\frac{1}{2}b\right)\left(\sqrt{3}\right)$

$b = \text{top} - \text{bottom}$

$b = (x+1) - (x^2 - 1) = -x^2 + x + 2$

$h = \frac{1}{2} b \sqrt{3} = \frac{1}{2}(-x^2 + x + 2)\sqrt{3}$

$= \frac{\sqrt{3}}{2}(-x^2 + x + 2)$

$A(x) = \frac{1}{2} b h$

$= \frac{1}{2}(-x^2 + x + 2)\left(\frac{\sqrt{3}}{2}(-x^2 + x + 2)\right) = \frac{\sqrt{3}}{4}(-x^2 + x + 2)^2$

$$\boxed{V = \int_a^b A(x)\,dx = \int_{-1}^{2} \left(\frac{\sqrt{3}}{4}(-x^2 + x + 2)^2\right) dx = 3.507 \text{ u}^3}$$

Answers to Worksheet on Volume by Cross Sections

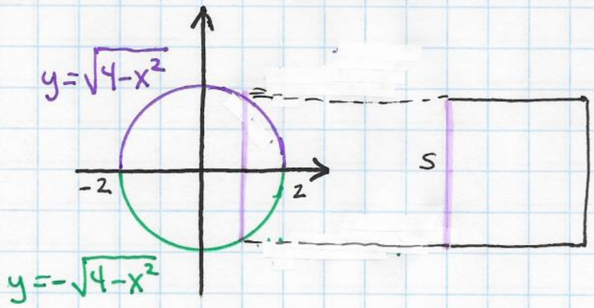
#2

(a)

Cross-sections are perpendicular to x-axis

Base is a circle: $x^2+y^2=4$

therefore $y=\pm\sqrt{4-x^2}$



cross-section is a square

side = top − bottom

$s=\sqrt{4-x^2}-\left(-\sqrt{4-x^2}\right)$

$s=2\sqrt{4-x^2}$

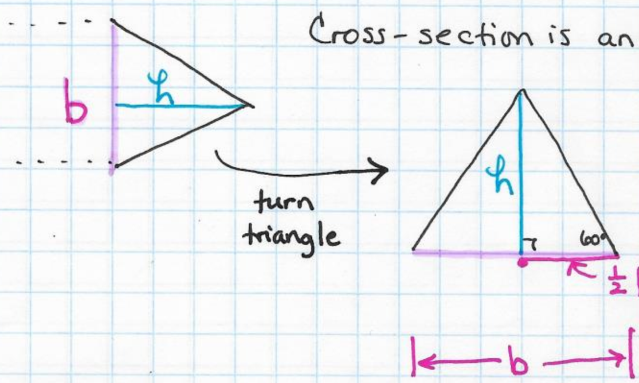
$A(x)=s^2$

$A(x)=\left(2\sqrt{4-x^2}\right)^2$

$$\boxed{V=\int_a^b A(x)\,dx=\int_{-2}^{2}\left(2\sqrt{4-x^2}\right)^2dx=42.667\ u^3}$$

(b)

Cross-section is an Equilateral Triangle.



$b=$ top − bottom $=2\sqrt{4-x^2}$ From part (a)

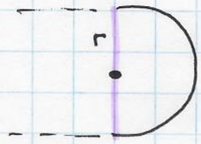
$h=\frac{1}{2}b\sqrt{3}=\frac{1}{2}\left(2\sqrt{4-x^2}\right)\sqrt{3}=\sqrt{3}\sqrt{4-x^2}$

$A(x)=\frac{1}{2}bh=\frac{1}{2}\left(2\sqrt{4-x^2}\right)\left(\sqrt{3}\sqrt{4-x^2}\right)=\sqrt{3}\left(4-x^2\right)$

$$\boxed{V=\int_a^b A(x)\,dx=\int_{-2}^{2}\left(\sqrt{3}\left(4-x^2\right)\right)dx=18.475\ u^3}$$

Answers to Worksheet on Volume by Cross Sections

(c)



Cross-sections are semi-circles!

radius $= \frac{1}{2}$ diameter $= \frac{1}{2}$ (top − bottom)

look at part a.

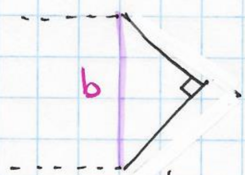
$$r = \frac{1}{2}\left(2\sqrt{4-x^2}\right)$$

$$r = \sqrt{4-x^2}$$

$$A(x) = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\sqrt{4-x^2}\right)^2 = \frac{1}{2}\pi\left(4-x^2\right)$$

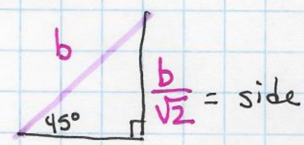
$$\boxed{V = \int_a^b A(x)\,dx = \int_{-2}^{2}\left(\frac{1}{2}\pi\left(4-x^2\right)\right)dx = 16.755\ u^3}$$

(d)



Cross-sections are isosceles right triangles (45°-45°-90°) with hypotenuse as base.

turn triangle



It's half a square!

b = top − bottom

$$b = 2\sqrt{4-x^2}$$

$$A(x) = \frac{1}{2}\left(\frac{b}{\sqrt{2}}\right)^2 = \frac{1}{2}\left(\frac{2\sqrt{4-x^2}}{\sqrt{2}}\right)^2 = 4-x^2$$

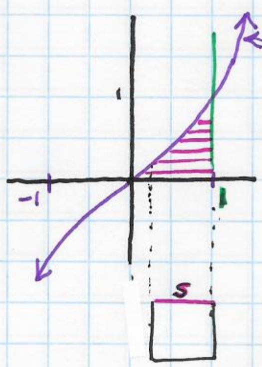
$$\boxed{V = \int_a^b A(x)\,dx = \int_{-2}^{2}\left(4-x^2\right)dx = 10.667}$$

Answers to Worksheet on Volume by Cross Sections

#3

* Cross-sections are perpendicular to y-axis!

$y = x^3 \rightarrow$ therefore $x = \sqrt[3]{y}$



ⓐ cross-sections are squares

$s = \text{right} - \text{left}$

$s = 1 - \sqrt[3]{y}$

$A(y) = s^2 = (1 - \sqrt[3]{y})^2$

$$V = \int_a^b A(y)\,dy = \int_0^1 (1 - \sqrt[3]{y})^2\,dy = 0.1 \ u^3$$

ⓑ cross-sections are semi-circles

diameter $= \text{right} - \text{left} = 1 - \sqrt[3]{y}$

radius $= \frac{1}{2}(1 - \sqrt[3]{y})$

$A(y) = \frac{1}{2}(\pi r^2) = \frac{\pi}{2}\left(\frac{1}{2}(1 - \sqrt[3]{y})\right)^2 = \frac{\pi}{8}(1 - \sqrt[3]{y})^2$

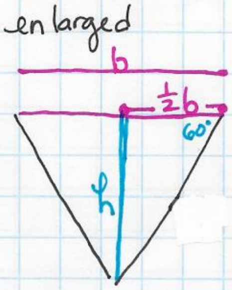
$$\boxed{V = \int_a^b A(y)\,dy = \int_0^1 \frac{\pi}{8}(1 - \sqrt[3]{y})^2\,dy = 0.039}$$

ⓒ Cross-sections are Equilateral triangles

$b = \text{right} - \text{left} = 1 - \sqrt[3]{y}$

$h = \frac{1}{2}(1 - \sqrt[3]{y})(\sqrt{3}) = \frac{\sqrt{3}}{2}(1 - \sqrt[3]{y})$

enlarged



$A(x) = \frac{1}{2}bh = \frac{1}{2}(1 - \sqrt[3]{y})\left(\frac{\sqrt{3}}{2}\right)(1 - \sqrt[3]{y})$

$A(x) = \frac{\sqrt{3}}{4}(1 - \sqrt[3]{y})^2$

$$\boxed{V = \int_0^1 \frac{\sqrt{3}}{4}(1 - \sqrt[3]{y})^2\,dy = 0.043 \ u^3}$$

special triangle:

$h = (\frac{1}{2}b)\sqrt{3}$