Step By Step #9 and #10 solutions

9 a. Average rate of change is $\frac{y-y_1}{x-x_1}$.

$f(1) = \frac{e+0}{1} = e$

$f(e) = \frac{e+1}{e^2}$

$$\frac{y-y_1}{x-x_1} = \frac{f(e)-f(1)}{e-1} = \frac{\frac{e+1}{e^2} - e}{e-1}$$

$$= \frac{\frac{e+1-e^3}{e^2}}{e-1} = \boxed{\frac{e+1-e^3}{e^2(e-1)}}$$

b. Equation of line tangent: Use point-slope $y-y_1 = m(x-x_1)$

$$f'(x) = \frac{x^2\left(\frac{1}{x}\right) - (e+\ln x)(2x)}{x^4}$$

$f'(1) = 1-2e$ slope

$f(1) = e$ point

$$\boxed{y-e = (1-2e)(x-1)}$$

c. Tangent line is horizontal when $f'(x) = 0$.

$$f'(x) = \frac{x-2x(e+\ln x)}{x^4} = \frac{1-2(e+\ln x)}{x^3} = 0$$

when $1-2(e+\ln x) = 0$

$\frac{1}{2} = e + \ln x$

$\ln x = \frac{1}{2} - e$

$\boxed{x = e^{\frac{1}{2}-e}}$

d. $\lim_{x\to 0^+} f(x)$. There is a Vertical Asymptote at $x=0$, so the limit will either be $+\infty$ or $-\infty$.

Test values

x	1	e
$f(x)$	e	$\frac{e+1}{e^2} \approx \frac{3.7}{7.4}$

$f(x)$ is decreasing from $x=0$

$$\boxed{\lim_{x\to 0^+} f(x) = \infty}$$

$\lim_{x\to\infty} f(x)$: Use L'Hôpital's Rule: $\lim_{x\to\infty} \frac{\frac{1}{x}}{2x} = \lim_{x\to\infty} \frac{1}{2x^2} = \boxed{0}$

10 a. $f'(x) = \left(\frac{1}{x+\frac{1}{x}}\right)\left(1 + \left(\frac{-1}{x^2}\right)\right) = \left(\frac{1}{\frac{x^2+1}{x}}\right)\left(\frac{x^2-1}{x^2}\right)$

$= \left(\frac{x}{x^2+1}\right)\left(\frac{x^2-1}{x^2}\right) = \boxed{\frac{x^2-1}{x^3+x}}$

b. Tangent line horizontal when $f'(x) = 0$.

$f'(x) = 0$ when numerator $= 0$: $x^2 - 1 = 0$

$x = \pm 1$

But $f(-1) = \text{DNE}$, so $\boxed{x = 1}$.

c. Equation of line tangent. Use point-slope form.

$f(2) = \ln\left(\frac{5}{2}\right)$ point

$f'(2) = \frac{3}{10}$ slope

$\boxed{y - \ln\frac{5}{2} = \frac{3}{10}(x-2)}$

d. $g'(x) = e^{2f(x)} \cdot 2f'(x)$

$g'(2) = e^{2f(2)} \cdot 2f'(2) = e^{2(\ln\frac{5}{2})} \cdot 2\left(\frac{3}{10}\right) = \left(\frac{5}{2}\right)^2\left(\frac{3}{5}\right) = \boxed{\frac{15}{4}}$

e. From part b: $f(x)$ has a horizontal tangent at $x = 1$.

Determine if $g'(1) = 0$:

$g'(x) = e^{2f(x)} \cdot 2f'(x)$

$g'(1) = e^{2f(1)} \cdot 2f'(1)$

$= e^{2\ln 2} \cdot 2(0) = \boxed{0}$

Therefore, $f(x)$ and $g(x)$ have horizontal tangent lines at the same x-value.