Derivatives of Exponential and Logarithm Functions

For problems 1 – 6 differentiate the given function.

1. $f(x)=2e^{x}-8^{x}$ $f'(x)=2e^{x}-8^{x}\ln 8$ Just use formula.

2. $g(t)=4\log_3(t)-\ln(t)$ $g'(t)=4\left(\frac{1}{t\ln 3}\right)-\frac{1}{t}=\frac{4}{t\ln 3}-\frac{1}{t}$ Just use formula.

3. $R(w)=(3^{w})(\log(w))$ $R'(w)=3^{w}\left(\frac{1}{w\ln 10}\right)+(\log w)(3^{w})(\ln 3)$ Formulas and Product Rule

4. $y=z^{5}-(e^{z})(\ln(z))$ $y'=5z^{4}-\left[(e^{z})\left(\frac{1}{z}\right)+(\ln z)(e^{z})\right]=5z^{4}-\frac{e^{z}}{z}-e^{z}\ln z$ Formulas and Product Rule

5. $h(y)=\frac{y}{1-e^{y}}$ $h'(y)=\frac{(1-e^{y})-y(-e^{y})}{(1-e^{y})^{2}}=\frac{1-e^{y}+ye^{y}}{(1-e^{y})^{2}}$ Formulas and Quotient Rule

6. $f(t)=\frac{1+5t}{\ln(t)}$ $f'(t)=\frac{(\ln t)(5)-(1+5t)\left(\frac{1}{t}\right)}{(\ln t)^{2}}=\frac{5\ln t-\frac{1}{t}+5}{(\ln t)^{2}}$ Formulas and Quotient Rule

7. Find the tangent line to $f(x)=7^{x}+4e^{x}$ at $x=0$.
Point: $x=0;\ y=7^{0}+4e^{0}=5$ slope: $f'(x)=7^{x}\ln 7+4e^{x}$ and $f'(0)=\ln 7+4$
$y-5=(\ln 7+4)x$

8. Find the tangent line to $f(x)=\ln(x)\log_2(x)$ at $x=2$.
Point: $f(2)=\ln 2\log_2 2=\ln 2$ slope: $f'(x)=\ln x\left(\frac{1}{x\ln 2}\right)+(\log_2 x)\left(\frac{1}{x}\right)$ $f'(2)=\frac{1}{2}+\frac{1}{2}=1$
$y-\ln 2=x-2$

9. Determine if $V(t)=\frac{t}{e^{t}}$ is increasing or decreasing at the following points. $V'(t)=\frac{e^{t}-te^{t}}{e^{2t}}=\frac{1-t}{e^{t}}=0$ when $t=1$

(a) $t=-4$ Inc (b) $t=0$ Inc (c) $t=10$ Dec

$V'(t)$: $++++\ 0\ ----$ (sign change at $t=1$)

10. Determine if $G(z)=(z-6)\ln(z)$ is increasing or decreasing at the following points.

(a) $z=1$ (b) $z=e^{5}$ (c) $z=e$

$G'(z)=(z-6)\left(\frac{1}{z}\right)+\ln z$

a) $G'(1)=-5+\ln 1=-5<0$, so dec

b) $G'(e)=\frac{e-6}{e}+\ln e=1-\frac{6}{e}+1<0$, so dec

c) $G'(e^{5})=\frac{e^{5}-6}{e^{5}}+\ln e^{5}=1-\frac{6}{e^{5}}+5>0$, so inc

Derivatives of Inverse Trig Functions

For each of the following problems differentiate the given function.

1. $T(z)=2\cos(z)+6\cos^{-1}(z)$