Calculus Name________________________________

Some Review

Use L'Hôpital's Rule to evaluate the limit if it can be applied. If it cannot be applied, write a * next to your answer.

1) $\lim_{x \to \infty} \frac{2x}{e^{2x}}$

2) $\lim_{x \to 3} \frac{x - 3}{\sqrt{x + 1} - 2}$

3) $\lim_{x \to 1} \frac{3 \ln x^2}{x^2 - 1}$

4) $\lim_{x \to 1} \frac{2 \ln x}{x - 1}$

5) $\lim_{x \to \infty} \frac{x}{\ln x}$

6) $\lim_{x \to \infty} \frac{\ln x}{x}$

7) $\lim_{x \to \infty} \frac{x}{e^{2x}}$

8) $\lim_{x \to 0} \frac{e^{2x} - 1}{4x}$

Differentiate each function with respect to *x*.

9) $y = \ln (x^5 + 5)^5$

10) $y = \ln \ln 4x^5$

11) $y = e^{(4x^3 + 1)^5}$

12) $y = e^{(3x^3 + 4)^4}$

13) $y = \ln 2x^3 \cdot e^{x^4}$

14) $y = e^{3x^4}(e^{4x^3} + 4)$

15) $y = \frac{e^{3x^2}}{e^{3x^3} + 1}$

16) $y = \frac{e^{4x^5}}{(x^2 - 2)^4}$

17) $y = \log_2 x^4$

18) $y = 3^{4x^2}$

19) $y = (3x^4 + 2)\log_2 x^2$

20) $y = (x^5 + 4)\log_3 x^2$

21) $y = \log_3 3x^4 \cdot (3x^5 + 5)$

22) $y = (2x^2 + 3)\log_5 5x^5$

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

23) $y = -e^{x-1}$ at $\left(-1, -\frac{1}{e^2}\right)$

24) $y = \ln (x + 1)$ at $(0, 0)$

25) $y = -\ln (x + 2)$ at $(0, -\ln 2)$

26) $y = -e^{-x-2}$ at $\left(0, -\frac{1}{e^2}\right)$

For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.

27) $y = -\ln (x + 2)$ at $(-1, 0)$

28) $y = -e^{x-2}$ at $\left(0, -\frac{1}{e^2}\right)$

For each problem, find the values of *c* that satisfy the Mean Value Theorem.

29) $y = x^2 - 8x + 17$; $[2, 4]$

30) $y = -x^3 + 4x^2 - 4$; $[0, 2]$

For each problem, determine if the Mean Value Theorem can be applied. If it can, find all values of *c* that satisfy the theorem. If it cannot, explain why not.

31) $y = \frac{x^2 - 4}{2x}$; $[1, 4]$

32) $y = \frac{x^2}{2x - 4}$; $[-2, 1]$

33) $y = -(7x + 42)^{\frac{2}{3}}$; $[-7, -4]$

For each problem, determine if Rolle's Theorem can be applied. If it can, find all values of *c* that satisfy the theorem. If it cannot, explain why not.

34) $y = -x^3 + x^2 + 4x + 1$; $[-2, 2]$

35) $y = \frac{x^2 + x - 6}{-x + 3}$; $[-3, 2]$

36) $y = \frac{x^2 - 1}{2x}$; $[-1, 1]$

37) $y = \frac{-x^2 + 3x + 18}{-x + 7}$; $[-3, 6]$

Answers to Some Review

1) 0
2) 4
3) 3
4) 2
5) ∞
6) 0
7) 0
8) $\frac{1}{2}$

9) $\frac{dy}{dx} = \frac{1}{(x^5+5)^5} \cdot 5(x^5+5)^4 \cdot 5x^4$
$= \frac{25x^4}{x^5+5}$

10) $\frac{dy}{dx} = \frac{1}{\ln 4x^5} \cdot \frac{1}{4x^5} \cdot 20x^4$
$= \frac{5}{x \ln 4x^5}$

11) $\frac{dy}{dx} = e^{(4x^3+1)^5} \cdot 5(4x^3+1)^4 \cdot 12x^2$
$= 60x^2 e^{(4x^3+1)^5} \cdot (4x^3+1)^4$

12) $\frac{dy}{dx} = e^{(3x^3+4)^4} \cdot 4(3x^3+4)^3 \cdot 9x^2$
$= 36x^2 e^{(3x^3+4)^4} \cdot (3x^3+4)^3$

13) $\frac{dy}{dx} = \ln 2x^3 \cdot e^{x^4} \cdot 4x^3 + e^{x^4} \cdot \frac{1}{2x^3} \cdot 6x^2$
$= \frac{e^{x^4}(4x^4 \ln 2x^3 + 3)}{x}$

14) $\frac{dy}{dx} = e^{3x^4} \cdot e^{4x^3} \cdot 12x^2 + \left(e^{4x^3} + 4\right) \cdot e^{3x^4} \cdot 12x^3$
$= 12x^2 e^{3x^4}\left(e^{4x^3} + xe^{4x^3} + 4x\right)$

15) $\frac{dy}{dx} = \frac{\left(e^{3x^3}+1\right) \cdot e^{3x^2} \cdot 6x - e^{3x^2} \cdot e^{3x^3} \cdot 9x^2}{\left(e^{3x^3}+1\right)^2}$
$= \frac{3xe^{3x^2}\left(2e^{3x^3} + 2 - 3xe^{3x^3}\right)}{\left(e^{3x^3}+1\right)^2}$

16) $\frac{dy}{dx} = \frac{(x^2-2)^4 \cdot e^{4x^5} \cdot 20x^4 - e^{4x^5} \cdot 4(x^2-2)^3 \cdot 2x}{\left((x^2-2)^4\right)^2}$
$= \frac{4xe^{4x^5}(5x^5 - 10x^3 - 2)}{(x^2-2)^5}$

17) $\frac{dy}{dx} = \frac{1}{x^4 \ln 2} \cdot 4x^3$
$= \frac{4}{x \ln 2}$

18) $\frac{dy}{dx} = 3^{4x^2} \ln 3 \cdot 8x$

19) $\frac{dy}{dx} = (3x^4+2) \cdot \frac{1}{x^2 \ln 2} \cdot 2x + \log_2 x^2 \cdot 12x^3$
$= \frac{2\left(6x^4 \log_2 x^2 \cdot \ln 2 + 3x^4 + 2\right)}{x \ln 2}$

20) $\frac{dy}{dx} = (x^5+4) \cdot \frac{1}{x^2 \ln 3} \cdot 2x + \log_3 x^2 \cdot 5x^4$
$= \frac{5x^5 \log_3 x^2 \cdot \ln 3 + 2x^5 + 8}{x \ln 3}$

21) $\frac{dy}{dx} = \log_3 3x^4 \cdot 15x^4 + (3x^5+5) \cdot \frac{1}{3x^4 \ln 3} \cdot 12x^3$
$= \frac{15x^5 \log_3 3x^4 \cdot \ln 3 + 12x^5 + 20}{x \ln 3}$

22) $\frac{dy}{dx} = (2x^2+3) \cdot \frac{1}{5x^5 \ln 5} \cdot 25x^4 + \log_5 5x^5 \cdot 4x$
$= \frac{4x^2 \log_5 5x^5 \cdot \ln 5 + 10x^2 + 15}{x \ln 5}$

23) $y = -\frac{1}{e^2} \cdot x - \frac{2}{e^2}$
24) $y = x$
25) $y = -\frac{1}{2}x - \ln 2$
26) $y = \frac{1}{e^2} \cdot x - \frac{1}{e^2}$
27) $y = x + 1$

28) $y = e^2x - \frac{1}{e^2}$

29) $\{3\}$

30) $\left\{\frac{2}{3}\right\}$

31) $\{2\}$

32) $\{0\}$

33) The function is not differentiable on $(-7, -4)$

34) $\left\{\frac{1 - \sqrt{13}}{3}, \frac{1 + \sqrt{13}}{3}\right\}$

35) $\{3 - \sqrt{6}\}$

36) The function is not continuous on $[-1, 1]$

37) $\{7 - \sqrt{10}\}$