Calculus Name_____



# Some Review for Fall Final

**For each problem, find the values of *c* that satisfy the Mean Value Theorem.**

1) $y = \dfrac{x^2}{3x + 6}$; $[-1, 2]$

**A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$, the acceleration function $a(t)$, the times $t$ when the particle changes directions, the intervals of time when the particle is moving left and moving right, and the intervals of time when the particle is slowing down and speeding up.**

2) $s(t) = -t^3 + 10t^2$

**For each problem, find all points of relative minima and maxima.**

3) $y = x^3 - 3x^2$

**For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.**

4) $y = -\sin(2x)$ at $(-\pi, 0)$

5) $y = -e^{x-1}$ at $\left(0, -\dfrac{1}{e}\right)$

**For each problem, find the open intervals where the function is concave up and concave down.**

6) $y = -\cos(2x)$; $[-\pi, \pi]$

**Evaluate each limit using L'Hôpital's Rule.**

7) $\lim\limits_{x \to 0} \dfrac{3(e^x - e^{-x})}{\sin x}$

8) $\lim\limits_{x \to \infty} \dfrac{\ln x^4}{\ln (x + 1)^5}$

**Solve each related rate problem.**

9) An observer stands 600 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the angle of elevation (in radians/sec) from the observer to rocket changing when the rocket is 800 ft from the ground?

**A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the displacement of the particle and the distance traveled by the particle over the given interval.**

10) $s(t) = t^2 - 9t - 90;\ 4 \leq t \leq 6$

**For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of $x$ and $y$.**

11) $-y + 1 = 3x^2 + 4y^2$

12) $5x^3 + 2x^2y^3 = -5y^2 + 4$

**Evaluate each limit.**

13) $\lim\limits_{x \to -\infty} -\dfrac{2x^2}{x^2 + 9}$

14) $\lim\limits_{x \to -\infty} \dfrac{2x^2}{3x - 2}$

15) $\lim\limits_{x \to -\infty} -e^{-3x}$

16) $\lim\limits_{x \to \infty} \dfrac{-x + 3}{x^2 + x + 1}$

**For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.**

17) $y = 2\cos(2x)$ at $\left(-\dfrac{3\pi}{4}, 0\right)$

18) $y = -2\cot(x)$ at $\left(\dfrac{\pi}{2}, 0\right)$

Calculus Name_____



# Some Review for Fall Final

**For each problem, find the values of *c* that satisfy the Mean Value Theorem.**

1) $y = \dfrac{x^2}{3x + 6}$; $[-1, 2]$

$\{0\}$

**A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$, the acceleration function $a(t)$, the times $t$ when the particle changes directions, the intervals of time when the particle is moving left and moving right, and the intervals of time when the particle is slowing down and speeding up.**

2) $s(t) = -t^3 + 10t^2$ $v(t) = -3t^2 + 20t$, $a(t) = -6t + 20$

Changes direction at: $t = \left\{\dfrac{20}{3}\right\}$, Moving left: $t > \dfrac{20}{3}$, Moving right: $0 < t < \dfrac{20}{3}$

Slowing down: $\dfrac{10}{3} < t < \dfrac{20}{3}$, Speeding up: $0 < t < \dfrac{10}{3}$, $t > \dfrac{20}{3}$

**For each problem, find all points of relative minima and maxima.**

3) $y = x^3 - 3x^2$

Relative minimum: $(2, -4)$
Relative maximum: $(0, 0)$

**For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.**

4) $y = -\sin(2x)$ at $(-\pi, 0)$

$y = -2x - 2\pi$

5) $y = -e^{x-1}$ at $\left(0, -\dfrac{1}{e}\right)$

$y = -\dfrac{1}{e} \cdot x - \dfrac{1}{e}$

**For each problem, find the open intervals where the function is concave up and concave down.**

6) $y = -\cos(2x)$; $[-\pi, \pi]$

Concave up: $\left(-\pi, -\dfrac{3\pi}{4}\right), \left(-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right), \left(\dfrac{3\pi}{4}, \pi\right)$ Concave down: $\left(-\dfrac{3\pi}{4}, -\dfrac{\pi}{4}\right), \left(\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right)$

**Evaluate each limit using L'Hôpital's Rule.**

7) $\lim\limits_{x \to 0} \dfrac{3(e^x - e^{-x})}{\sin x}$

6

8) $\lim\limits_{x \to \infty} \dfrac{\ln x^4}{\ln (x + 1)^5}$

$\dfrac{4}{5}$

  

**Solve each related rate problem.**

9) An observer stands 600 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the angle of elevation (in radians/sec) from the observer to rocket changing when the rocket is 800 ft from the ground?

$a$ = altitude of rocke

Equation: $\tan\theta = \frac{a}{60}$

$\left.\frac{d\theta}{dt}\right|_{a=800} = \frac{1}{600\sec^2}$

**A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the displacement of the particle and the distance traveled by the particle over the given interval.**

10) $s(t) = t^2 - 9t - 90;\ 4 \leq t \leq 6$

Displacement: 2

Distance traveled: $\frac{5}{2} = 2.5$

**For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of $x$ and $y$.**

11) $-y + 1 = 3x^2 + 4y^2$

$\frac{dy}{dx} = \frac{6x}{-1 - 8y}$

12) $5x^3 + 2x^2y^3 = -5y^2 + 4$

$\frac{dy}{dx} = \frac{-15x^2 - 4xy^3}{6y^2x^2 + 10y}$

**Evaluate each limit.**

13) $\lim_{x \to -\infty} -\frac{2x^2}{x^2 + 9}$

$-2$

14) $\lim_{x \to -\infty} \frac{2x^2}{3x - 2}$

$-\infty$

15) $\lim_{x \to -\infty} -e^{-3x}$

$-\infty$

16) $\lim_{x \to \infty} \frac{-x + 3}{x^2 + x + 1}$

$0$

**For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.**

17) $y = 2\cos(2x)$ at $\left(-\frac{3\pi}{4}, 0\right)$

$y = \frac{1}{4}x + \frac{3\pi}{16}$

18) $y = -2\cot(x)$ at $\left(\frac{\pi}{2}, 0\right)$

$y = -\frac{1}{2}x + \frac{\pi}{4}$