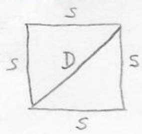
Calculus

Name *Related Rates #2*

Solve each related rate problem.

1) A hypothetical square shrinks at a rate of 18 m²/min. At what rate are the diagonals of the square changing when the diagonals are 4 m each?

$\boxed{-\frac{9}{2}\ m/min}$



$\frac{dA}{dt} = -18\ m^2/min$

Find $\frac{dD}{dt}$ when $D = 4m$

$A = s^2$

Need to rewrite s^2 in terms of D:

$s = \frac{D}{\sqrt{2}}$

$A = \left(\frac{D}{\sqrt{2}}\right)^2 = \frac{1}{2}D^2$

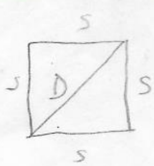
$\frac{dA}{dt} = D\frac{dD}{dt}$

$-18 = 4\frac{dD}{dt}$

$\boxed{\frac{dD}{dt} = -\frac{9}{2}\ m^2/min}$

2) A hypothetical square grows so that the length of its diagonals are increasing at a rate of 6 m/min. How fast is the area of the square increasing when the diagonals are 4 m each?

$\boxed{24\ m/min}$



$\frac{dD}{dt} = 6\ m/min$

Find $\frac{dA}{dt}$ when $D = 4m$

$A = s^2$

Need to rewrite s^2 in terms of D:

$s = \frac{D}{\sqrt{2}}$

$A = \left(\frac{D}{\sqrt{2}}\right)^2 = \frac{1}{2}D^2$

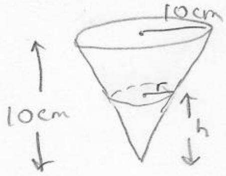
$\frac{dA}{dt} = D\frac{dD}{dt}$

$\frac{dA}{dt} = (4)(6)$

$= \boxed{24\ m/min}$

3) A conical paper cup is 10 cm tall with a radius of 10 cm. The bottom of the cup is punctured so that the water leaks out at a rate of $\frac{16\pi}{3}$ cm³/sec. At what rate is the water level changing when the water level is 7 cm?

$\boxed{\frac{-16}{147}\ \text{cm/sec}}$

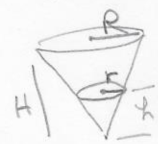


$\frac{dV}{dt} = -\frac{16\pi}{3}\ \text{cm}^3/\text{sec}$

Find $\frac{dh}{dt}$ when $h = 7$cm

$V = \frac{1}{3}\pi r^2 h$

Need to rewrite Volume in terms of h only



$\frac{r}{h} = \frac{R}{H}$

In this case:

$\frac{r}{h} = \frac{10}{10}$, so $r = h$

$\rightarrow V = \frac{1}{3}\pi h^2 h$

$V = \frac{1}{3}\pi h^3$

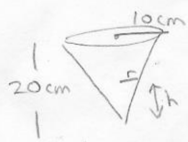
$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$

$-\frac{16\pi}{3} = 49\pi \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-16}{147}\ \text{cm/sec}}$

4) A conical paper cup is 20 cm tall with a radius of 10 cm. The bottom of the cup is punctured so that the water level goes down at a rate of 4 cm/sec. At what rate is the volume of water in the cup changing when the water level is 5 cm?

$\boxed{-25\pi\ \text{cm}^3/\text{sec}}$



$\frac{dh}{dt} = -4\text{cm/sec}$

Find $\frac{dV}{dt}$ when $h = 5$cm

$\frac{10}{20} = \frac{r}{h} \quad r = \frac{1}{2}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$

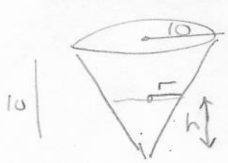
$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{4}\pi(25)(-4) = \boxed{-25\pi\ \text{cm}^3/\text{sec}}$

5) A conical paper cup is 10 cm tall with a radius of 10 cm. The bottom of the cup is punctured so that the water level goes down at a rate of 4 cm/sec. At what rate is the volume of water in the cup changing when the water level is 9 cm?

$\boxed{-324\pi\ \text{cm}^3/\text{sec}}$



$\frac{dh}{dt} = -4\text{cm/sec}$

Find $\frac{dV}{dt}$ when $h = 9$ cm

$\frac{10}{10} = \frac{r}{h} \rightarrow r = h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (h^2) h$

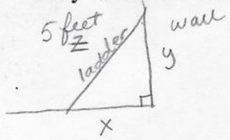
$V = \frac{1}{3}\pi h^3$

$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \pi(81)(-4) = \boxed{-324\pi\ \text{cm}^3/\text{sec}}$

6) A 5 ft ladder is leaning against a wall and sliding towards the floor. The top of the ladder is sliding down the wall at a rate of 8 ft/sec. How fast is the base of the ladder sliding away from the wall when the base of the ladder is 3 ft from the wall?

$\boxed{\frac{32}{3}\ \text{ft/sec}}$



Pythagorean Theorem

$$z^2 = x^2 + y^2$$

Substitute constant values:

$$25 = x^2 + y^2$$

Take Derivative:

$$0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

Substitute and solve:

$$0 = 2(3)\frac{dx}{dt} + 2(4)(-8)$$

$$\frac{dx}{dt} = \frac{64}{6} = \boxed{\frac{32}{3}\ \text{ft/sec}}$$

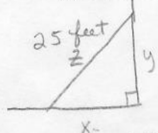
constant:
$z = 5$ feet

changing:
$\frac{dy}{dt} = -8$ ft/sec

Find $\frac{dx}{dt}$ when
$x = 3$ feet
this means: $25 = 9 + y^2$
$y = 4$ feet

7) A 25 ft ladder is leaning against a wall and sliding towards the floor. The top of the ladder is sliding down the wall at a rate of 9 ft/sec. How fast is the base of the ladder sliding away from the wall when the base of the ladder is 24 ft from the wall?

$\boxed{\frac{21}{8}\ \text{ft/sec}}$



constant: $z = 25$ ft

changing
$\frac{dy}{dt} = -9$ ft/sec
Find $\frac{dx}{dt}$
when $x = 24$ feet
so $25^2 = 24^2 + y^2$
$y = 7$ feet

$$z^2 = x^2 + y^2$$

$$25^2 = x^2 + y^2$$

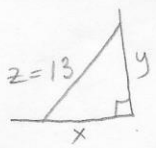
$$0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$0 = 2(24)\frac{dx}{dt} + 2(7)(-9)$$

$$\frac{dx}{dt} = \frac{126}{48} = \boxed{\frac{21}{8}\ \text{ft/sec}}$$

8) A 13 ft ladder is leaning against a wall and sliding towards the floor. The foot of the ladder is sliding away from the base of the wall at a rate of $\frac{2}{x}$ ft/sec, where x is the distance from the base of the ladder to the wall. How fast is the top of the ladder sliding down the wall when the top of the ladder is 12 ft from the ground?

$\boxed{-\frac{1}{6}\ \text{ft/sec}}$



Constant $z = 13$ feet

changing
$\frac{dx}{dt} = \frac{2}{x}$ ft/sec
Find $\frac{dy}{dt}$ when
$y = 12$ ft
so $x = \sqrt{13^2 - 12^2} = 5$

$$13^2 = x^2 + y^2$$

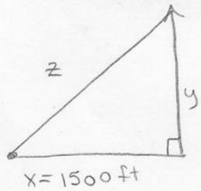
$$0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$0 = 2(5)\left(\frac{2}{5}\right) + 2(12)\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{-4}{24} = \boxed{-\frac{1}{6}\ \text{ft/sec}}$$

9) An observer stands 1500 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 400 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 800 ft from the ground?

$\boxed{\frac{3200}{17} \text{ ft/sec}}$



$z^2 = x^2 + y^2$

Substitute constant values

$z^2 = 1500^2 + y^2$

Derivative

$2z \frac{dz}{dt} = 2y \frac{dy}{dt}$

$2(1700) \frac{dz}{dt} = 2(800)(400)$

$\frac{dz}{dt} = \frac{640{,}000}{3400}$

$\boxed{\frac{dz}{dt} = \frac{3200}{17} \text{ ft/sec}}$

<u>Constant</u>

$x = 1500$ ft

<u>Changing</u>

$\frac{dy}{dt} = 400$ ft/sec

Find $\frac{dz}{dt}$ when

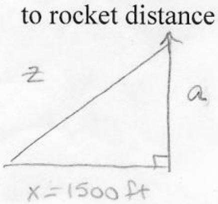
$y = 800$ ft

so $z = \sqrt{1500^2 + 800^2}$

$= 1700$ ft

10) An observer stands 1500 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of $\frac{80000}{a}$ ft/sec, where a is the altitude of the rocket. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 800 ft from the ground?

$\boxed{\frac{800}{17} \text{ ft/sec}}$



$z^2 = x^2 + a^2$

$z^2 = 1500^2 + a^2$

$2z \frac{dz}{dt} = 2a \frac{da}{dt}$

$2(1700) \frac{dz}{dt} = 2(800)\left(\frac{80{,}000}{800}\right)$

$\boxed{\frac{dz}{dt} = \frac{800}{17} \text{ ft/sec}}$

<u>Constant</u>

$x = 1500$ ft

<u>Changing</u>

$\frac{da}{dt} = \frac{80{,}000}{a}$ ft/sec

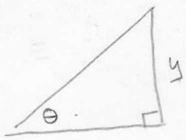
Find $\frac{dz}{dt}$ when

$a = 800$ ft, so $\frac{da}{dt} = \frac{80{,}000}{800}$ ft/sec

$z = 1700$ ft

11) An observer stands 600 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 500 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the angle of elevation (in radians/sec) from the observer to rocket changing when the rocket is 800 ft from the ground?

$\boxed{\frac{3}{10} \text{ rads/sec}}$



x=600ft

Constant

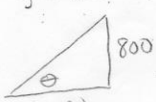
x = 600 ft

changing

$\frac{dy}{dt} = 500$ ft/sec

Find $\frac{d\theta}{dt}$ when

y = 800 ft



$\theta = \tan^{-1}\left(\frac{800}{600}\right)$
$= 0.927$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{y}{600}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{600}\frac{dy}{dt}$

$\frac{d\theta}{dt} = \frac{1}{600}(500)\cos^2(0.927)$

$\boxed{\frac{d\theta}{dt} = 0.30023 \text{ rads/sec}}$ due to rounding

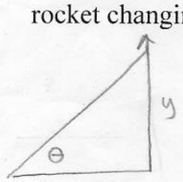
*To avoid rounding error:

$\frac{d\theta}{dt} = \frac{1}{600}(500)\left(\cos^2\left(\tan^{-1}\frac{800}{600}\right)\right)$

$\boxed{\frac{d\theta}{dt} = 0.3 \text{ rads/sec}}$

12) An observer stands 400 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 200 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the angle of elevation (in radians/sec) from the observer to rocket changing when the rocket is 300 ft from the ground?

$\boxed{\frac{8}{25} \text{ rads/sec}}$



x= 400ft

Constant

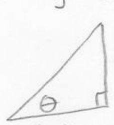
x= 400 ft

Changing

$\frac{dy}{dt} = 200$ ft/sec

Find $\frac{d\theta}{dt}$ when

y = 300 ft



$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{y}{400}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{400}\frac{dy}{dt}$

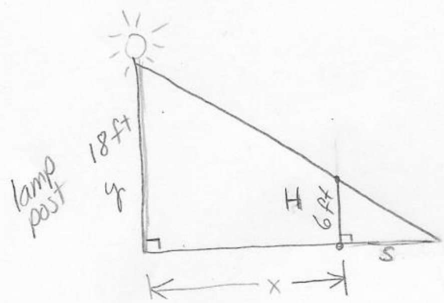
$\frac{d\theta}{dt} = \frac{1}{400}\frac{dy}{dt}\cos^2\theta$

$\frac{d\theta}{dt} = \frac{1}{400}(200)\left(\cos\left(\tan^{-1}\frac{3}{4}\right)\right)^2$

$\boxed{\frac{d\theta}{dt} = \frac{8}{25} \text{ rads/sec}}$

13) A 6 ft tall person is walking away from a 18 ft tall lamppost at a rate of $\frac{6}{x}$ ft/sec, where x is the distance from the person to the lamppost. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 15 ft from the lamppost?

$\boxed{\frac{1}{5} \text{ ft/sec}}$



constant

$y = 18$ ft

$H = 6$ ft

changing

$\frac{dx}{dt} = \frac{6}{x}$ ft/sec

Find $\frac{ds}{dt}$ when $x = 15$ ft,

If $x = 15$,

$\frac{dx}{dt} = \frac{6}{15} = \frac{2}{5}$

$\frac{18}{x+s} = \frac{6}{s}$

$18s = 6(x+s)$

$18s = 6x + 6s$

$12s = 6x$

$2s = x$

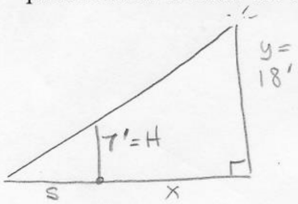
$2\frac{ds}{dt} = \frac{dx}{dt}$

$2\frac{ds}{dt} = \frac{2}{5}$

$\boxed{\frac{ds}{dt} = \frac{1}{5} \text{ ft/sec}}$

14) A 7 ft tall person is walking towards a 18 ft tall lamppost at a rate of $\frac{2}{x}$ ft/sec, where x is the distance from the person to the lamppost. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 12 ft from the lamppost?

$\boxed{-\frac{7}{66} \text{ ft/sec}}$



$$\frac{s}{7} = \frac{s+x}{18}$$

$$18s = 7s + 7x$$

$$11s = 7x$$

$$11\frac{ds}{dt} = 7\frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{7}{11}\left(-\frac{1}{6}\right)$$

$$\boxed{\frac{ds}{dt} = -\frac{7}{66} \text{ ft/sec}}$$

constant
$y = 18$ ft
$H = 7$ ft

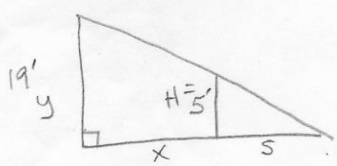
changing
$\frac{dx}{dt} = -\frac{2}{x}$ ft/sec

Find $\frac{ds}{dt}$ when
$x = 12$ feet

If $x = 12$,
$\frac{dx}{dt} = -\frac{1}{6}$ ft/sec

15) A 5 ft tall person is walking away from a 19 ft tall lamppost at a rate of $\frac{2}{x}$ ft/sec, where x is the distance from the person to the lamppost. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 14 ft from the lamppost?

$\boxed{\frac{5}{98} \text{ ft/sec}}$



$$\frac{19}{x+s} = \frac{5}{s}$$

$$19s = 5x + 5s$$

$$14s = 5x$$

$$14\frac{ds}{dt} = 5\frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{5}{14}\left(\frac{1}{7}\right)$$

$$\boxed{\frac{ds}{dt} = \frac{5}{98} \text{ ft/sec}}$$

constant
$y = 19$ feet
$H = 5$ feet

changing
$\frac{dx}{dt} = \frac{2}{x}$ ft/sec

when $x = 14$ ft

so $\frac{dx}{dt} = \frac{2}{14} = \frac{1}{7}$ ft/sec

Find $\frac{ds}{dt}$