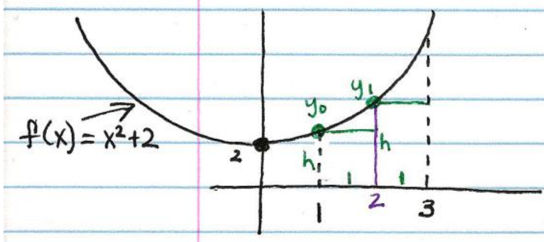
Unit 5.1 – Rectangular Approximation Methods

Left Rectangular Approximation Method — LRAM

$f(x) = x^2 + 2$ on $[1, 3]$ with $n = 2$

↳ n = number intervals



There are 2 intervals.
Each interval has width: $\frac{3-1}{2} = \boxed{1}$

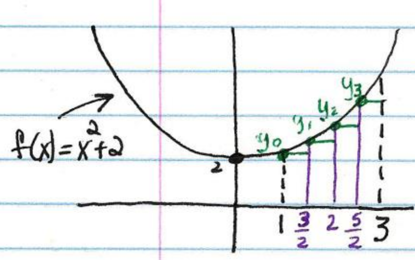
LRAM uses the left points. In other words, the height of the rectangle is on the left side.

$$\text{LRAM} = \left(\frac{b-a}{n}\right)(y_0 + y_1)$$

$y_0 = f(1) = 1^2 + 2 = 3$
$y_1 = f(2) = 2^2 + 2 = 6$

$$= 1(3 + 6)$$
$$= \boxed{9}$$

$f(x) = x^2 + 2$ on $[1, 3]$ with $n = 4$

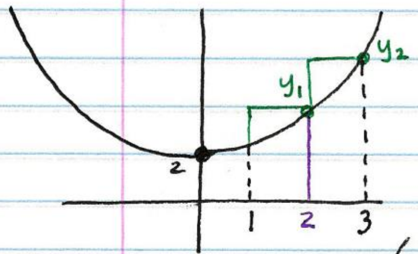


$y_0 = f(1) = 1^2 + 2 = 3$
$y_1 = f(\frac{3}{2}) = (\frac{3}{2})^2 + 2 = \frac{17}{4}$
$y_2 = f(2) = 2^2 + 2 = 6$
$y_3 = f(\frac{5}{2}) = (\frac{5}{2})^2 + 2 = \frac{33}{4}$

$$\text{LRAM} = \left(\frac{b-a}{n}\right)(y_0 + y_1 + y_2 + y_3)$$
$$= \left(\frac{3-1}{4}\right)(y_0 + y_1 + y_2 + y_3)$$
$$= \frac{1}{2}\left(3 + \frac{17}{4} + 6 + \frac{33}{4}\right)$$
$$= \frac{1}{2}\left(21\tfrac{1}{2}\right) = \boxed{10.75}$$

Right Rectangular Approximation Method - RRAM

$f(x) = x^2 + 2$ on $[1, 3]$ with $n = 2$



RRAM uses the right points.
In other words, the height of the rectangle is on the right side.

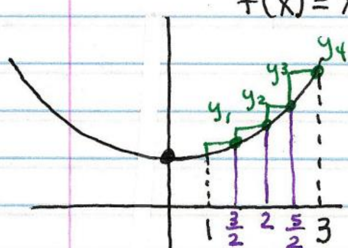
$$RRAM = \left(\frac{b-a}{n}\right)(y_1 + y_2)$$

$$= \frac{3-1}{2}(6 + 11)$$

$$= \boxed{17}$$

$y_1 = f(2) = 6$
$y_2 = f(3) = 11$

$f(x) = x^2 + 2$ on $[1, 3]$ with $n = 4$



$$RRAM = \left(\frac{b-a}{n}\right)(y_1 + y_2 + y_3 + y_4)$$

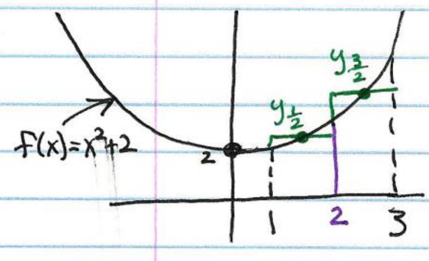
$$= \left(\frac{3-1}{4}\right)\left(\frac{17}{4} + 6 + \frac{33}{4} + 11\right)$$

$$= \frac{1}{2}(29.5) = \boxed{14.75}$$

$y_1 = f(\frac{3}{2}) = \frac{17}{4}$
$y_2 = f(2) = 6$
$y_3 = f(\frac{5}{2}) = \frac{33}{4}$
$y_4 = f(3) = 11$

Midpoint Rectangular Approximation Method - MRAM

$f(x) = x^2 + 2$ on $[1, 3]$ with $n = 2$



MRAM uses the midpoint of the interval as the height

$$\text{MRAM} = \left(\frac{b-a}{n}\right)\left(y_{\frac{1}{2}} + y_{\frac{3}{2}}\right)$$

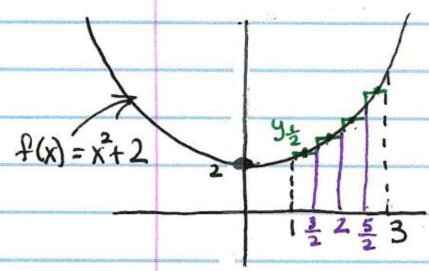
$$= \frac{3-1}{2}\left(\frac{17}{4} + \frac{33}{4}\right)$$

$$= 1\left(\frac{17}{4} + \frac{33}{4}\right) = \boxed{12.5}$$

$y_{\frac{1}{2}} = f\left(\frac{3}{2}\right) = \frac{17}{4}$

$y_{\frac{3}{2}} = f\left(\frac{5}{2}\right) = \frac{33}{4}$

$f(x) = x^2 + 2$ on $[1, 3]$ with $n = 4$



$y_{\frac{1}{2}} = f\left(\frac{5}{4}\right) = \frac{57}{16}$

$y_{\frac{3}{2}} = f\left(\frac{7}{4}\right) = \frac{81}{16}$

$$\text{MRAM} = \frac{b-a}{n}\left(y_{\frac{1}{2}} + y_{\frac{3}{2}} + y_{\frac{5}{2}} + y_{\frac{7}{2}}\right)$$

$$= \frac{3-1}{4}\left(\frac{57}{16} + \frac{81}{16} + \frac{113}{16} + \frac{153}{16}\right)$$

$$= \frac{1}{2}(25.25) = \boxed{12.625}$$

$y_{\frac{5}{2}} = f\left(\frac{9}{4}\right) = \frac{113}{16}$

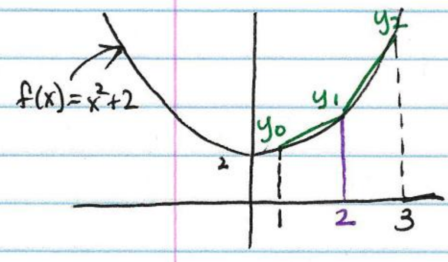
$y_{\frac{7}{2}} = f\left(\frac{11}{4}\right) = \frac{153}{16}$

Unit 5.5

Trapezoidal Approximations

Trapezoidal Rule $= \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n\right)$

$f(x) = x^2 + 2$ on $[1, 3]$ with $n = 2$



h is the width of the interval

so $h = \frac{b-a}{n}$

Here, $h = 1$

$y_0 = f(1) = 3$

$y_1 = f(2) = 6$

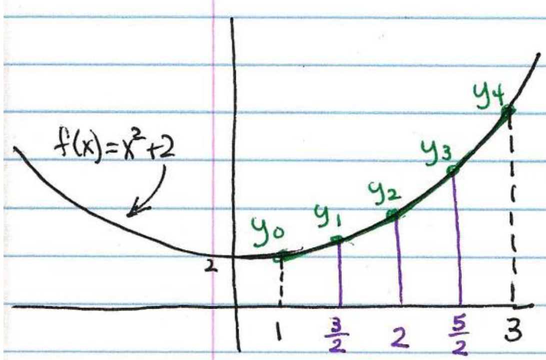
$y_2 = f(3) = 11$

Trapezoidal Appx $= \frac{h}{2}\left(y_0 + 2y_1 + y_2\right)$

$= \frac{1}{2}\left(3 + 2(6) + 11\right)$

$= \frac{1}{2}(26) = \boxed{13}$

$f(x) = x^2 + 2$ on $[1, 3]$ with $n = 4$



TrapAppx $= \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right)$

$y_0 = f(1) = 3$

$y_1 = f(\frac{3}{2}) = \frac{17}{4}$

$y_2 = f(2) = 6$

$y_3 = f(\frac{5}{2}) = \frac{33}{4}$

$y_4 = f(3) = 11$

TAM $= \frac{.5}{2}\left(3 + 2\left(\frac{17}{4}\right) + 2(6) + 2\left(\frac{33}{4}\right) + 11\right)$

$= \boxed{12.75}$