2: integrand
-1 each error
Note: 0/2 if integral is not of form $c\int_a^b (R^2 - r^2)\,dx$

(1)

$f(x) = 2x(1-x)$
$g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$

(a) Area Shaded Region $= \int_0^1 \left((2x(1-x)) - (3(x-1)\sqrt{x}) \right) dx = \boxed{1.133}$

1: integral
1: answer

(b) $R = 2 - g(x) = 2 - 3(x-1)\sqrt{x}$
$r = 2 - f(x) = 2 - 2x(1-x)$

1: limits, const
2: integrand
1: answer

$V = \pi \int_0^1 \left((2 - 3(x-1)\sqrt{x})^2 - (2 - 2x(1-x))^2 \right) dx = \boxed{16.179}$

(c) $h(x) = kx(1-x)$ for $0 \le x \le 1$

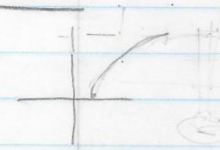
$h(x) - g(x) = \text{side}$
$A(x) = s^2$

2: integrand
1: answer

$\int_0^1 (h(x) - g(x))^2\, dx = 15$

$\boxed{\int_0^1 \left(kx(1-x) - 3(x-1)\sqrt{x} \right)^2 dx = 15}$

(2)



(a) $A = \int_1^{10} \left(\sqrt{x-1} \right) dx = 18$

1: limits
1: integrand
1: answer

(b) $R = 3 - 0 \quad r = 3 - \sqrt{x-1}$
$V = \pi \int_1^{10} \left(3^2 - (3 - \sqrt{x-1})^2 \right) dx = \boxed{212.058}$

1: limits, const
1: integrand
1: answer

(c) $y = \sqrt{x-1}$, therefore $x = y^2 + 1$ $\quad 10 = y^2 + 1$
$R = 10 - (y^2+1) \quad r = 0 \quad y = 3$

$V = \pi \int_0^3 \left((10 - (y^2+1))^2 \right) dy = \boxed{407.150}$

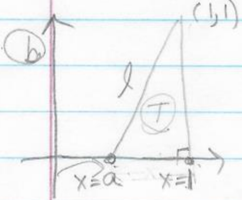
1: limits, const
1: integrand
1: answer

No calc

3

(a) $\int_0^1 x^n\,dx = \frac{1}{n+1}\left(x^{n+1}\right)\Big|_0^1 = \frac{1}{n+1}(1-0) = \boxed{\frac{1}{n+1}}$

(b)



Slope of $l = \frac{\text{rise}}{\text{run}} = \frac{1}{x-a}\Big|_{x=1} = \frac{1}{1-a}$ = slope of l at $x=1$

$y = x^n$

$y' = nx^{n-1}\Big|_{x=1} = n$ = slope of l at $x=1$

$\frac{1}{1-a} = n$

$\frac{1}{n} = 1-a$

$a = 1-\frac{1}{n} \longrightarrow$

Area $\Delta_T = \frac{1}{2}(1-a)(1)$

$= \frac{1}{2}\left(1-\left(1-\frac{1}{n}\right)\right)$

$= \frac{1}{2}\left(\frac{1}{n}\right) = \boxed{\frac{1}{2n}}$

(c) Area $S = \int_0^1 x^n\,dx - \left(\frac{1}{2n}\right) = \boxed{\frac{1}{n+1} - \frac{1}{2n}}$

Maximize area of S: $\frac{dA}{dt} = 0$

$A = \frac{1}{n+1} - \frac{1}{2n}$

$\frac{dA}{dn} = \frac{-1}{(n+1)^2} + \frac{1}{2n^2} = 0$

$\frac{1}{2n^2} = \frac{1}{(n+1)^2}$

$2n^2 = (n+1)^2$

$\sqrt{2}\,n = n+1$

$n(\sqrt{2}-1) = 1$

$n = \frac{1}{\sqrt{2}-1} \cdot \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}$

$\boxed{n = \frac{\sqrt{2}+1}{2-1} = \sqrt{2}+1}$

Calc

4

a. $A_R = \int_0^{0.178} \left(4^{-x} - \left(\frac{1}{4} + \sin(\pi x)\right)\right) dx = \boxed{0.065}$

b. $A_S = \int_{0.178}^{1} \left(\frac{1}{4} + \sin(\pi x) - 4^{-x}\right) dx = \boxed{0.410}$

c. $R = \frac{1}{4} + \sin(\pi x) - (-1)$ $\quad r = 4^{-x} - (-1)$

$$V = \pi \int_{0.178}^{1} \left(\left(\frac{1}{4} + \sin(\pi x) + 1\right)^2 - \left(4^{-x} + 1\right)^2\right) dx = \boxed{4.559}$$

5

Calc

a. $Area_R = \int_0^{1.136} \left(1 + \sin(2x) - e^{\frac{x}{2}}\right) dx = \boxed{0.429}$

b. $R = 1 + \sin(2x) - 0$ $\quad r = e^{\frac{x}{2}} - 0$

$$V = \pi \int_0^{1.136} \left(\left(1 + \sin(2x)\right)^2 - \left(e^{\frac{x}{2}}\right)^2\right) dx = \boxed{4.267}$$

c.

$r = \frac{1}{2}\left(f(x) - g(x)\right)$ $\quad V = \int_0^{1.136} \frac{\pi}{2}\left(\frac{1}{2}\left(1 + \sin(2x) - e^{\frac{x}{2}}\right)\right)^2 dx$

$= \boxed{0.078}$

calc

6

(a) $\text{Area}_R = \int_{0.159}^{3.146} (\ln x - (x-2))\,dx = \boxed{1.949}$

(b) $R = \ln x - (-3) = \ln x + 3$ $\quad r = x-2-(-3) = x+1$

$V = \pi \int_{0.159}^{3.146} \left((\ln x+3)^2 - (x+1)^2 \right) dx = \boxed{34.199}$

(c) $y = \ln x$ $\quad y = x-2$

$x = e^y$ $\quad x = y+2$ $\quad R = y+2-0$

$r = e^y - 0$

$$\boxed{V = \pi \int_{-1.841}^{1.146} \left((y+2)^2 - (e^y)^2 \right) dy}$$

1

a. $\text{Area}_R = \int_{-1.373}^{0} \left(\frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x - 0 \right) dx = \boxed{2.903}$

b. $R = f(x) - (-2) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x + 2$

$r = 0 - (-2) = 2$

$V = \pi \int_{-1.373}^{0} \left(\left(\frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x + 2 \right)^2 - 4 \right) dx = \boxed{59.361}$

c. $f'(0) = -0.5$ point $(0,3)$ tangent line

$y - 3 = -0.5x$

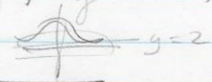
$y = -0.5x + 3$

$\boxed{\text{Area}\,S = \int_{0}^{3.39} \left(-0.5x + 3 - \left(\frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x \right) \right) dx}$

calc

(8)

$y = \frac{20}{1+x^2} \qquad y = 2 \qquad (-3, 2) \qquad (3,2)$

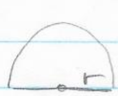


ⓐ $\text{Area}_R = \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right) dx = \boxed{37.962}$

ⓑ $R = \frac{20}{1+x^2} - 0 \qquad r = 2-0$

$V = \pi \int_{-3}^{3} \left(\left(\frac{20}{1+x^2}\right)^2 - 2^2 \right) dx = \boxed{1871.190}$

ⓒ



$r = \frac{1}{2}\left(\frac{20}{1+x^2} - 2\right)$

$V = \int_{-3}^{3} \frac{1}{2}\pi \left(\frac{1}{2}\left(\frac{20}{1+x^2} - 2 \right)\right)^2 dx$

$= \boxed{174.268}$

(9) ⓐ $\text{Area } R = \int_{0.446}^{1.554} \left(e^{2x-x^2} - 2 \right) dx = \boxed{0.514}$

ⓑ $\text{Area } S = \int_0^2 \left(e^{2x-x^2} - 1 \right) dx -$ Area of Region R =

$= 2.060 - 0.514 = \boxed{1.546}$

ⓒ $\boxed{V = \pi \int_{0.446}^{1.554} \left(\left(e^{2x-x^2} - 1\right)^2 - 1^2 \right) dx}$

$R = e^{2x-x^2} - 1$

$r = 2-1$

Calc.

10.

(a) Area R $= \int_0^2 \left(\sin\pi x - (x^3-4x) \right) dx = \boxed{4}$

(b) $\boxed{\text{Area} = \int_{0.788}^{1.675} \left(-2 - (x^3-4x) \right) dx} = .724$

(c) $A(x) = \left(\sin\pi x - (x^3-4x)\right)^2$

$$V = \int_0^2 \left(\left(\sin\pi x - (x^3-4x) \right)^2 \right) dx = \boxed{9.978}$$

(d) Area Cross Section $= (3-x)\left(\sin\pi x - (x^3-4x) \right)$

$$V = \int_0^2 \left((3-x)\left(\sin\pi x - (x^3-4x) \right) \right) dx = \boxed{8.370}$$

11. Calc

(a) $y = \sqrt{x}$, $y = \frac{x}{3}$

$$\text{Area} = \int_0^9 \left(\sqrt{x} - \frac{x}{3} \right) dx = \boxed{4.5}$$

(b) $y = \sqrt{x}$ $\quad y = \frac{x}{3}$

$x = y^2$ $\quad x = 3y$ $\qquad V = \pi \int_0^3 \left((3y-(-1))^2 - (y^2-(-1))^2 \right) dy$

$$= \boxed{130.062}$$

(c) $A(y) = s^2$

$A(y) = (3y-y^2)^2$

$3y-y^2$

$$V = \int_0^3 \left((3y-y^2)^2 \right) dy = \boxed{8.1}$$

12. $y = 2x \quad y = x^2$

(a) $\text{Area} = \int_0^2 (2x - x^2)\,dx = \left(x^2 - \frac{1}{3}x^3\right)\Big|_0^2 = 4 - \frac{8}{3} = \boxed{\frac{4}{3}}$

(b) $A(x) = \sin\left(\frac{\pi}{2}x\right)$

$\text{Area} = \frac{2}{\pi}\int_0^2 \frac{\pi}{2}\left(\sin\left(\frac{\pi}{2}x\right)\right)dx = \frac{2}{\pi}\left(-\cos\left(\frac{\pi}{2}x\right)\right)\Big|_0^2$

$= \frac{2}{\pi}\left(-(-1) + 1\right) = \boxed{\frac{4}{\pi}}$

(c) $y = 2x \quad y = x^2$

$x = \frac{1}{2}y \quad x = \sqrt{y}$

$\boxed{V = \int_0^4 \left(\sqrt{y} - \frac{1}{2}y\right)^2 dy}$

13.

(a) $\text{Area} = \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right)dx = \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{4}\right)\Big|_0^4 = \frac{16}{3} - 4 = \boxed{\frac{4}{3}}$

(b) $\sqrt{x} - \frac{x}{2}$

$V = \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right)^2 dx = \int_0^2 \left(x - x^{\frac{3}{2}} + \frac{1}{4}x^2\right)dx$

$\left(\sqrt{x} - \frac{x}{2}\right)^2 = x - x\sqrt{x} + \frac{1}{4}x^2$

$= \left(\frac{1}{2}x^2 - \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{12}x^3\right)\Big|_0^4$

$= \left(\frac{1}{2}(16) - \frac{2}{5}(32)\right) + \frac{64}{12} = 8 - \frac{64}{5} + \frac{16}{3}$

$= \frac{120 - 192 + 80}{15} = \boxed{\frac{8}{15}}$

(c) $\boxed{V = \pi\int_0^4 \left(\left(2 - \frac{x}{2}\right)^2 - \left(2 - \sqrt{x}\right)^2\right)dx}$

$R = 2 - \frac{x}{2}$

$r = 2 - \sqrt{x}$