Function Analysis + Particle Motion FRQs

1. a. $h(x)$ has a horizontal tangent when $h'(x)=0$.

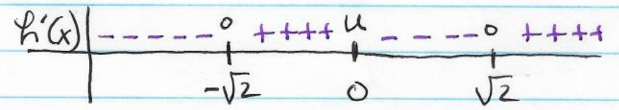
$h(x)$ has a local maximum when $h'(x)$ changes from positive to negative.

$h(x)$ has a local minimum when $h'(x)$ changes from negative to positive.

$h'(x) = \frac{x^2-2}{x} = 0$ $\qquad$ $x^2-2=0$

$x = \pm\sqrt{2}$

$h(x)$ has horizontal tangents at $x = \pm\sqrt{2}$.



$h(x)$ has no local maximum.
$h(x)$ has local minimums at $x = \pm\sqrt{2}$.

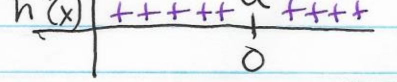
b. $h(x)$ is concave up when $h''(x) > 0$.

$h'(x) = \frac{x^2-2}{x}$ $\qquad$ $h''(x) = \frac{x(2x) - (x^2-2)(1)}{x^2} = \frac{x^2+2}{x^2}$

$h''(x) = \frac{x^2+2}{x^2} = 0$ $\qquad$ $x^2+2=0$ $\qquad$ $x^2=0$

$x^2 \neq -2$! $\qquad$ $x=0$

h'' und. @ $x=0$



$h(x)$ is concave up for all $x \neq 0$.

c. $h(4) = -3$
$h'(4) = \frac{7}{2}$

$\boxed{y + 3 = \frac{7}{2}(x-4)}$

d. At $x=4$, the graph of $h(x)$ is concave up, therefore the equation of the tangent line at $x=4$ lies below $h(x)$.

2. a. Absolute maximum occurs when f' changes from positive to negative or at an endpoint.

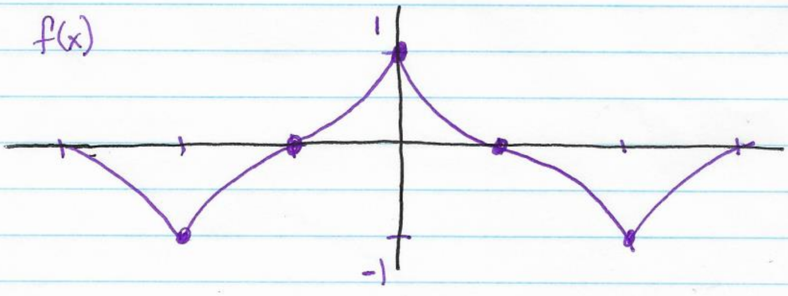
$\boxed{\text{Absolute maximum at } x=0.}$

Absolute minimum occurs when f' changes from negative to positive or at an endpoint.

$\boxed{\text{Absolute minimum at } x=2.}$

b. $f(x)$ has an inflection point when the sign of f'' changes. This occurs at $x=1$ <u>and</u> at $x=-1$ because $f(x)$ is an even function.

c. $f(x)$



3. a. $x(t) = (t-2)^3(t-6)$

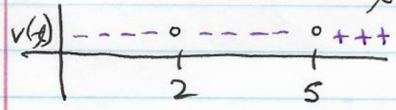
$x'(t) = v(t) = (t-2)^3 + (t-6)(3)(t-2)^2$

$= (t-2)^3 + (3t-18)(t-2)^2 = 0$

$= (t-2)^2\left((t-2) + (3t-18)\right) = 0$

$= (t-2)^2(4t-20) = 0$

$t = 2, 5$



Particle moves to right when $v(t) > 0$: on interval $(5, \infty)$.

b. Particle is at rest when $v(t) = 0$: At $t = 2, 5$.

c. Particle changes direction when $v(t)$ changes signs: At $t = 5$.

d. Particle moves to left between $t=0$ and $t=5$ and then changes direction and moves to the right. The particle is farthest left at $t=5$. $x(5) = 3^3(-1) = -27$.

4. a. $x = \frac{1}{3}t^3 - 3t^2 + 8t$

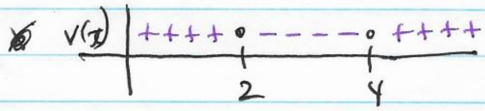
Particle moves to right when $x'(t) = v(t) > 0$.

$x'(t) = v(t) = t^2 - 6t + 8$

At $t=0$: v(0) = 8 which is greater than zero, therefore the particle is moving to the right at $t=0$.

b. Particle moves to left when $v(t) < 0$.

$v(t) = t^2 - 6t + 8$
$= (t-4)(t-2) = 0$
$t = 2, 4$



Particle moves to left over interval (2, 4).

c. $x(3) = \frac{1}{3}(27) - 3(9) + 24 = \boxed{6}$

d. Total distance

t	$x(t)$	
0	0	$^{20}/_3$
2	$^{20}/_3$	
3	6	$^{2}/_3$

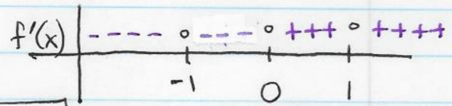
$\frac{20}{3} + \frac{2}{3} = \boxed{\frac{22}{3} = \text{Total Distance}}$

5. a. $f(x)$ increasing when $f'(x) > 0$.

$f'(x) = 3(x^2-1)^2(2x) = 0$

$6x(x^2-1)^2 = 0$

$x = 0, \pm 1$



f(x) increasing over interval $(0, \infty)$.

b. $f(x)$ has relative minima when $f'(x)$ changes from negative to positive.

$f(x)$ has relative maxima when $f'(x)$ changes from positive to negative.

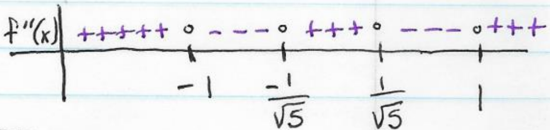
Relative minima at $x = 0$. There are no relative maximums.

c. $f(x)$ concave down when $f''(x) < 0$ and concave up when $f''(x) > 0$.

$f''(x) = (6x)(2)(x^2-1)(2x) + 6(x^2-1)^2$

$= (x^2-1)(30x^2-6) = 0$

$x = \pm 1, \pm \frac{1}{\sqrt{5}}$



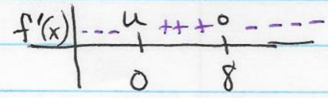
$f(x)$ concave up over intervals $(-\infty, -1)$, $\left(-\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$, $(1, \infty)$.

6. a. f(x) increasing when $f'(x)>0$.

$$f'(x) = 8x^{-\frac{1}{3}} - 4 = \frac{8}{\sqrt[3]{x}} - 4 = 0$$

$$8 = 4\sqrt[3]{x}$$

$$x = 8$$



Not defined at x=0

f(x) increasing on interval (0,8).

b. f(x) has a relative max at (8,16).

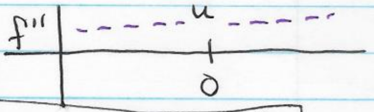
$$f(8) = 12(8)^{2/3} - 4(8) = 16$$

c. $f(0)=0$

f(x) has a relative min at (0,0).

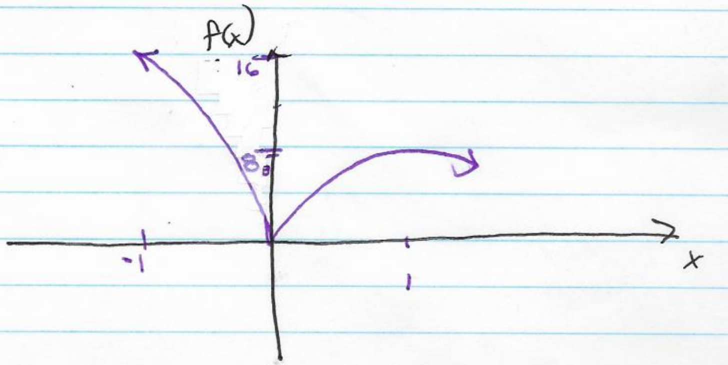
d. $f''(x) = -\frac{8}{3}x^{-\frac{4}{3}} = 0$

Not defined at x=0.



f(x) concave down on intervals (-∞,0), (0,∞).

e.



Note: $f(1) = 12-4=8$

$f(-1) = 12+4 = 16$