Function Analysis Free Response

$\boxed{\#1}$

ⓐ f has horizontal tangent when $f'(x)=0$.
This occurs at $x=-7,-1,4,8$.

ⓑ f has a relative maxima when $f'(x)$ changes from positive to negative. This occurs at $x=-1, 8$.

ⓒ f is concave down when the slope of $f'(x)$ is negative (where $f'(x)$ is decreasing). This occurs in the intervals $(-3,2)$ and $(6,10)$.

$\boxed{\#2}$

ⓐ f attains a relative minimum when $f'(x)$ changes from negative to positive. This occurs at $x=-1$.

ⓑ f attains a relative maximum when $f'(x)$ changes from positive to negative. This occurs at $x=-5$.

ⓒ $f''(x)<0$ when $f'(x)$ is decreasing.
This occurs in the intervals $(-7,-3)$, $(2,3)$ and $(3,5)$.
Note: (3 is not included because there is a vertical tangent at $x=3$, therefore $f'(x)$ is not differentiable at $x=3$).

ⓓ f attains its absolute maximum at an endpoint or at its relative maxima at $x=-5$.
$f(-5)>f(-7)$ because $f(x)$ is increasing on $(-7,-5)$ ($f'(x)>0$ there).
The graph shows that the net change from $x=-5$ to $x=7$ is positive, so the absolute maximum is at $x=7$.

#3

(a) Points of inflection occur when f' changes from increasing to decreasing at $x=1$ and when f' changes from decreasing to increasing at $x=3$.

(b) • Absolute minimum occurs at an endpoint or at the relative minima at $x=4$. Because the graph of f decreases from $x=-1$ to $x=4$, and then increases from $x=4$ to $x=5$, the absolute minimum must occur at $x=4$.

• The absolute maximum must occur at $x=-1$ or $x=5$ because there are no relative maximas in the interval $[-1,5]$.

$\int_{-1}^{5} f'(t)\,dt = f(5)-f(-1) < 0$ (more area <u>under</u> the x-axis)

Because $f(5)-f(-1)<0$,
then $f(5) < f(-1)$
so the absolute maximum occurs at $x=-1$.

(c) $g(x) = x f(x)$

$g'(x) = x f'(x) + f(x)$

$g'(2) = 2(f'(2)) + f(2)$

(Note: find $f'(2)$ on the graph!) → $= 2(-1) + 6$

$= 4$

slope of the tangent $= 4$

(Note: this info. given) point: $g(2) = 2f(2) = 2(6) = 12$

$\boxed{y - 12 = 4(x-2)}$

#4

$f(0)=5 \quad f'(x)=e^{-\frac{x}{4}} \sin(x^2)$

(a) f' is decreasing on the interval $(1.7, 1.9)$ so f is concave down on the interval.

(b) There is a relative maxima when $f'(x)=0$. This occurs at $x=1.772$. The absolute maximum will occur at an endpoint or at $x=1.772$. Because $f'>0$ on the interval $(0, 1.772)$, f is increasing on the interval, therefore $f(0) < f(1.772)$.

$$f(1.772) = f(0) + \int_0^{1.772} f'(x)\,dx = 5 + \int_0^{1.772} f'(x)\,dx$$
$$= 5.679$$

$$f(3) = f(0) + \int_0^3 f'(x)\,dx = 5 + \int_0^3 f'(x)\,dx$$
$$= 5.579$$

This shows that f has an absoute maximum at $x = 1.772$.

(c) $y-y_1 = m(x-x_1) \qquad m = f'(2) = -0.459$

$$f(2) = f(0) + \int_0^2 f'(x)\,dx = 5.623$$

$$\boxed{y - 5.623 = -0.459(x-2)}$$