Area/Volume Free Response

calculator

1. $f(x) = x^4 - 2.3x^3 + 4$, $y = 4$ – graph to find intersections $(0, 2.3)$

ⓐ $V = \pi \int_0^{2.3} \left(6^2 - (x^4 - 2.3x^3 + 6)^2\right) dx$ $= \boxed{98.868}$

$R = 4 - (-2) = 6$

$r = (x^4 - 2.3x^3 + 4) - (-2)$
$= x^4 - 2.3x^3 + 6$

2: Integrand
1: limits
1: answer

ⓑ Cross section: $V = \int_a^b A(x)\,dx$

Area of Isosceles right triangle with leg on R.

2: integrand
1: answer

$4 - (x^4 - 2.3x^3 + 4)$

$\text{leg} = -x^4 + 2.3x^3$

$A(x) = \frac{1}{2} b h$
$= \frac{1}{2}(-x^4 + 2.3x^3)^2$

$V = \int_0^{2.3} \frac{1}{2}(-x^4 + 2.3x^3)^2 dx = \boxed{3.574}$

ⓒ Area Region $= \int_0^{2.3} (4 - (-x^4 - 2.3x^3 + 4))\,dx$

$= \int_0^{2.3} (4 - f(x))\,dx$

$$\boxed{\int_0^k (4 - f(x))\,dx = \int_k^{2.3} (4 - f(x))\,dx}$$

1: area of one region

1: equation

No calculator

2. $f(x) = 2x^2 - 6x + 4 \qquad g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$

ⓐ Area of $R = \int_0^2 \left(g(x) - f(x)\right) dx$

1: integrand
2: antiderivatives
1: answer

$$= \int_0^2 \left(4\cos\left(\tfrac{1}{4}\pi x\right) - \left(2x^2 - 6x + 4\right)\right) dx$$

$u = \frac{1}{4}\pi x$

$du = \frac{1}{4}\pi\, dx$

$$= (4)\left(\tfrac{4}{\pi}\right)\int_0^2 \left(\cos\left(\tfrac{\pi}{4}x\right)\right)\left(\tfrac{\pi}{4}dx\right) - \int_0^2 \left(2x^2 - 6x + 4\right) dx$$

$$= \tfrac{16}{\pi}\left(\sin \tfrac{\pi}{4}x\right)\Big|_0^2 - \left(\tfrac{2}{3}x^3 - 3x^2 + 4x\right)\Big|_0^2$$

$$= \tfrac{16}{\pi}\left(\sin\tfrac{\pi}{2} - \sin 0\right) - \left(\tfrac{2}{3}(8) - 3(4) + 4(2) - 0\right)$$

$$= \tfrac{16}{\pi} - \tfrac{16}{3} + 12 - 8 = \boxed{\tfrac{16}{\pi} - \tfrac{4}{3}}$$

ⓑ $R = 4 - (2x^2 - 6x + 4) \qquad r = 4 - 4\cos\left(\frac{1}{4}\pi x\right)$

2: integrand
1: limits and constant (π)

$$V = \pi \int_0^2 \left[\left(4 - (2x^2 - 6x + 4)\right)^2 - \left(4 - 4\cos\left(\tfrac{1}{4}\pi x\right)\right)^2\right] dx$$

ⓒ Cross section ⊥ x-axis is square.

$V = \int_a^b A(x)\, dx \qquad A(x) = s^2$

$$A(x) = \left(4\cos\left(\tfrac{\pi}{4}x\right) - (2x^2 - 6x + 4)\right)^2$$

$$V = \int_0^2 \left(4\cos\left(\tfrac{\pi}{4}\right) - (2x^2 - 6x + 4)\right)^2 dx$$

1: integrand
1: limits

3. $f(x) = \sqrt{x}$ $\quad g(x) = 6 - x$

ⓐ Area $R = \int_0^4 (\sqrt{x} - 0)\,dx + \int_4^6 ((6-x) - 0)\,dx$

$= \int_0^4 x^{\frac{1}{2}}\,dx + \int_4^6 (6-x)\,dx$

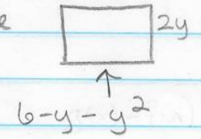
$= \frac{2}{3} x^{\frac{3}{2}} \Big|_0^4 + \left(6x - \frac{1}{2}x^2\right)\Big|_4^6 = \frac{16}{3} + 36 - 18 - (24 - 8)$

$= \frac{16}{3} + 2 = \boxed{\frac{22}{3}}$

1: integral
1: antiderivative
1: answer

ⓑ Cross section $\perp$ y-axis!

Rectangle



$y = g(x)$ $\quad y = f(x)$

$y = 6 - x$ $\quad y = \sqrt{x}$

$x = 6 - y$ $\quad x = y^2$

$A(x) = 2y(6 - y - y^2)$ $\quad \boxed{V = \int_0^2 \left(2y(6-y-y^2)\right) dy}$

2: integrand
1: answer

ⓒ Want to know when slope of $f(x) \perp$ slope $g(x)$

Slope of $g(x) = -1$

so, Slope of $f(x) = 1$. Find when $f'(x) = 1$:

$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = 1$

$\frac{1}{\sqrt{x}} = 2$

$x = \frac{1}{4} \rightarrow f\left(\frac{1}{4}\right) = \frac{1}{2}$

$\boxed{\text{The point } P \text{ is } \left(\frac{1}{4}, \frac{1}{2}\right).}$

1: $f'(x)$
1: equation
1: answer

No calculator

4. $y = 2\sqrt{x}$, $y = 6$, y-axis.

(a) Area $R = \int_0^9 (6 - 2\sqrt{x})\,dx = \left(6x - \frac{4}{3}x^{\frac{3}{2}}\right)\Big|_0^9 = \boxed{18}$

1: integrand
1: antideriv.
1: answer

(b) Washer: $R = 7 - 2\sqrt{x}$, $r = 7 - 6 = 1$

$$\boxed{V = \pi \int_0^9 \left((7-2\sqrt{x})^2 - 1^2\right)dx}$$

2: integrand
1: limits and constant

(c) Cross section $\perp$ y-axis

$y = 2\sqrt{x}$
$x = \frac{1}{4}y^2$

$3\left(\frac{1}{4}y^2\right)$

$\frac{1}{4}y^2$

$V = \int_a^b A(x)\,dx \qquad A(x) = \left(\frac{1}{4}y^2\right)\left(\frac{3}{4}y^2\right)$

$$\boxed{V = \int_0^6 \left(\left(\frac{1}{4}y^2\right)\left(\frac{3}{4}y^2\right)\right)dy}$$

2: integrand
1: answer