Thursday 5/10 FRQ Review Solutions

1. a. $P'(32) = \$65.92/\text{kg}$

The company's profit is increasing when it sells 32 kg of chili powder because $P'(32) > 0$.

b. Determine where $P'(x) = 0$

$P'(x) = 48 + 2.8x - 0.14x^{1.8} = 0$ when $x = 58.358$

Check endpoints and relative maximum.

x	P(x)
0	-270
58.358	2893
80	1873.3

The company must sell 58.358 kg of chili powder to maximize its weekly profit.

c. Rate of Sales $= S(t) = 2 + \cos\left(\frac{\pi}{10} t^2\right)$ kg/hour

Amt Chili powder $= \int_0^5 S(t)\,dt = 11.433$ kg

d. $S'(3) = -0.582$ kg/hr^2

The rate of sales of chili powder is decreasing at 0.582 kg/hr^2 at $t = 3$ hours.

2\. a. Avg Rate of Change $= \frac{G(10) - G(6)}{10 - 6} = \frac{2100 - 900}{4}$

$= 300$ games / week^2

b. $\int_0^{12} G(t)\,dt \approx 3(450) + 3(900) + 4(2100) + 2(2400) = 17{,}250$ games

$\int_0^{12} G(t)\,dt$ represents the total number of games sold between $t=0$ and $t=12$ weeks.

c. Rate of sales of games for $t > 12$ = Amt of sales at $t=12$ + 100 (number of weeks after 12 weeks)

$= 2400 + 100(t - 12)$

Number of games sold between $t=12$ and $t=20$ $= \int_{12}^{20} \left(2400 + 100(t-12)\right) dt = 22{,}400$ games

d. Total number games sold $= \int_{12}^{20} m(t)\,dt = 15{,}784.077$

Based on this model, 15,784 games will be sold between $t=12$ weeks and $t=20$ weeks.

3. a. $\frac{f(5)-f(0)}{5-0} = \frac{0-2}{-5} = \frac{2}{5}$

b. $f(t) = g'(x)$

g has a point of inflection when $g'(x) = f(x)$ changes from positive slope to negative slope or from negative slope to positive slope.
This occurs at $x = -3$ and $x = 0$.

c. $\frac{1}{c+5} \int_{-5}^{c} f(x)\,dx = \frac{1}{2}$

$\frac{1}{c+5}\left(-\pi - 1 + 2 + c\right) = \frac{1}{2}$

$-2\pi + 2 + 2c = c + 5$

$c = 3 + 2\pi$

d. $h(x) = f\left(\frac{1}{2}x\right)$

$h'(x) = \frac{1}{2} f'\left(\frac{1}{2}x\right)$

$h'(6) = \frac{1}{2} f'\left(\frac{1}{2}(6)\right) = \frac{1}{2} f'(3) = \frac{1}{2}\left(\frac{-2}{c}\right) = \boxed{-\frac{1}{c}}$

4.

a. Remember $3\sin x = 3$

when $\sin x = 1$

$x = \pi/2$

$$\text{Area} = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (3 - 3\sin x)\,dx = (3x + 3\cos x)\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \frac{3\pi}{2} + 0 - \left(\pi + \frac{3}{2}\right)$$

$$= \frac{3\pi}{2} - \pi - \frac{3}{2}$$

$$= \boxed{\frac{\pi}{2} - \frac{3}{2}}$$

b. Area with respect to $k = A = \int_{k}^{\pi/2} (3 - 3\sin x)\,dx$

Rate of change of Area $= A' = \frac{d}{dx}\int_{k}^{\frac{\pi}{2}} (3 - 3\sin x)\,dx$

Using 2nd FTC: $= (0 - (3 - 3\sin k))$

$= -3 + 3\sin k$

$$A'\left(\frac{\pi}{6}\right) = -3 + 3\sin\frac{\pi}{6} = -3 + 3\left(\frac{1}{2}\right) = \boxed{-\frac{3}{2}}$$

c. $R = 5 - \sin x$

$r = 5 - 3 = 2$

$$V = \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left((5 - \sin x)^2 - (2)^2\right) dx$$

5\. a. $W(t)$ = Temp. in refrigerator

point $(0, 3°C)$

slope = $\frac{dW}{dt}$ at 0 hours $(0, 3°C)$ = $\frac{3\cos t}{2W} = \frac{3\cos 0}{2(3)} = \frac{1}{2}$

$$\boxed{\begin{aligned} W - 3 &= \tfrac{1}{2}(t - 0) \\ W &= \tfrac{1}{2}t + 3 \end{aligned}}$$

$W(0.4) = \frac{1}{2}(0.4) + 3 = \boxed{3.2°C}$

b. $\frac{dW}{dt} = \frac{3\cos t}{2W}$

$W\,dW = \frac{3}{2}\cos t\,dt$

$\frac{1}{2}W^2 = \frac{3}{2}\sin t + C$

at $(0, 3)$: $\frac{9}{2} = 0 + C$

$\frac{1}{2}W^2 = \frac{3}{2}\sin t + \frac{9}{2}$

$W^2 = 3\sin t + 9$

$W = \sqrt{3\sin t + 9}$ for $0 \le t \le 24$ hours

c. $W(t)$ = Temp. in refrigerator

$20°C$ = Constant temp. in kitchen

$20 - W(t)$ = difference in the two temps.

Rate of change of cost = \$0.001/hour/degree temp in kitchen exceeds temp in refrigerator

Cost = $\$0.001 \int_0^{24} (20 - w(t))\,dt$

6 a. f is continuous at $x=1$ because

$$\lim_{x\to 1^-} f(x)=7, \quad \lim_{x\to 1^+} f(x)=7, \quad f(1)=7$$

By the definition of continuity, $f(x)$ is continuous at $x=1$.

b. Check endpoints and any relative minimums and maximums in the interval $[-2,2]$

$$f'(x)=\begin{cases}-2-2x, & x\le 1\\ 4e^{x-1}, & x>1\end{cases} \qquad \begin{aligned} f'(x)&=0=-2-2x\\ x&=-1\\ f'(x)&=0\neq 4e^{x-1}\end{aligned}$$

At $x=1$: left hand derivative $=-4$

right hand derivative $=4$

The function f is not differentiable at $x=1$.

x	$f(x)$
-2	$10+4-4=10$
-1	$10+2-1=11$
1	$10-2-1=7$
2	$3+4e$

The absolute minimum of f occurs at $f(1)=7$.

The absolute maximum of f occurs at $f(2)=3+4e$

c. $$\int_0^2 f(x)\,dx = \int_0^1 (10-2x-x^2)\,dx + \int_1^2 (3+4e^{x-1})\,dx$$

$$=\left(10x-x^2-\tfrac{1}{3}x^3\right)\Big|_0^1 + \left(3+4e^{x-1}\right)\Big|_1^2$$

$$=10-1-\tfrac{1}{3}+\left(6+4e-(3+4)\right)$$

$$=8-\tfrac{1}{3}+4e$$

$$=\boxed{\tfrac{23}{3}+4e}$$