FRQ Test

Part I

1. a. $a(5.1) = v'(5.1) = 6.492$ — 1: answer

b. speed $= |v(t)| = 1$ when $t = 0.772$ and $t = 1.401$ — 1: considers $|v(t)|=1$

(Graph $y = |v(t)|$ and $y = 1$ and use calculator to find intersection. — 1: answer

The speed of the particle is 1 when $t = 0.772$ and $t = 1.401$.

c. position at $t=4 = s(4) = 7 + \int_0^4 v(t)\,dt = 6.712$ — 1: integral; 1: uses initial cond

The particle is moving away from the origin because $s(4) > 0$ and $v(4) = 1.213 > 0$. — 1: position; 1: movement with sign

d. $s(4) = 6.712$
$s(0) = 7$
$s(1) = 7 + \int_0^1 v(t)\,dt = 9.404$

Yes. $s(t)$ is continuous because $v(t) = s'(t)$, so $s(t)$ is differentiable.

At $t=0$, position $s(0) = 7$ — 1: sandwiches initial condition
$t=1$, position $s(1) = 9.404$ — 1: answer with reason
$t=4$, position $s(4) = 6.712$

By the Intermediate Value Theorem, the particle must equal 7 between $t=1$ and $t=4$.

2. ① a. $G'(8) \approx \dfrac{G(10)-G(6)}{10-6} = \dfrac{2100-900}{4} = 300$ games/week^2

1: approximation

③ b. $\int_0^{12} G(t)\,dt \approx 3(450)+3(900)+4(2100)+2(2400) = 17{,}250$ games

$\int_0^{12} G(t)\,dt$ represents the total number of games sold between 0 and 12 weeks.

1: RRAM
1: approximation
1: explanation

③ c. Rate of sales of games for $t>12$ = Amt of sales at $t=12$ + 100 (number weeks <u>after</u> 12 weeks)

$= 2400 + 100(t-12)$

1: rate function
1: integral
1: answer

Number games sold between $t=12$ and $t=20$ weeks $= \int_{12}^{20} \left(2400+100(t-12)\right) dt = \boxed{22{,}400 \text{ games}}$

② d. Total games sold $= \int_{12}^{20} M(t)\,dt = 15{,}784.077$

1: integral

1: answer

Based on this model, 15,784 games will be sold between $t = 12$ weeks and $t = 20$ weeks.

3\. (3) a. $\frac{g(5)-g(-5)}{5-(-5)} = \frac{12-(7+\pi)}{10} = \boxed{\frac{5-\pi}{10}}$

1: difference quotient
2: answer

$g(5) = \int_1^5 f(t)\,dt = 12$
$g(-5) = \int_1^{-5} f(t)\,dt = 1+\pi+6 = 7+\pi$

(1) b. $g'(3) = f(3) = 4$

1: answer

The instantaneous rate of change of g at $x=3$ is 4.

(2) c. The graph of g is concave up when $g'(x) = f(x)$ is increasing.

2: Intervals and jya

This occurs on $(-5,2)$ and $(0,3)$.

(3) d. g has a critical point when $g'(x) = f(x) = 0$
This occurs at $x=-2$ and $x=1$.

$x=-2$ is neither a local minimum nor a local maximum because g' does not change signs.

$x=1$ is a local minimum because g' changes from negative to positive.

1: considers $f(x)=0$
1: CP at $x=-2, 1$
1: answers with jya

4, 3 a. point $(2,-8)$

slope at point $= \left.\frac{3x^2}{y}\right|_{(2,-8)} = \frac{3(2)^2}{-8} = -\frac{3}{2}$

1: slope

1: tangent line equation

1: approximation

$$\boxed{y+8 = -\frac{3}{2}(x-2)}$$

$$y = -\frac{3}{2}(x-2)-8$$

$$f(1.8) \approx -\frac{3}{2}(-.2)-8 = \boxed{-7.7}$$

6 b. $\frac{dy}{dx} = \frac{3x^2}{y}$

$$y\,dy = 3x^2\,dx$$

$$\tfrac{1}{2}y^2 = x^3 + C$$

$$y^2 = 2x^3 + C$$

at $(2,-8)$: $64 = 16 + C$

$C = 48$

$$y^2 = 2x^3 + 48$$

$$\boxed{y = -\sqrt{2x^3+48}} \quad \text{for } x > -\sqrt[3]{24}$$

1: separate variables

2: antiderivatives

1: "+c"

1: uses initial cond.

1: solves for y

5. ② a. Area $R = \int_0^{\frac{\pi}{2}} \left(g(x) - f(x)\right)dx = \int_0^{\frac{\pi}{2}} \left(e^x - \sqrt{\cos x}\right) dx$

1: integrand
1: limits of integral

④ b. $R = g(x)$
$r = f(x)$

$$V = \pi \int_0^{\frac{\pi}{2}} \left((g(x))^2 - (f(x))^2 \right) dx$$

$$V = \pi \int_0^{\pi/2} \left((e^x)^2 - (\sqrt{\cos x})^2 \right) dx$$

$$= \pi \int_0^{\frac{\pi}{2}} \left(e^{2x} - \cos x\right) dx$$

$$= \pi \left(\frac{1}{2} e^{2x} - \sin x \right) \Big|_0^{\frac{\pi}{2}}$$

$$= \pi \left(\left(\frac{1}{2} e^{\pi} - 1\right) - \left(\frac{1}{2} - 0\right) \right) = \pi \left(\frac{1}{2} e^{\pi} - \frac{3}{2} \right)$$

$$= \boxed{\frac{\pi e^{\pi} - 3\pi}{2}}$$

1: integrand
2: antiderivative
1: answer

③ c. Semicircle Area $= \frac{1}{2}\pi r^2$

$$r = \frac{1}{2}\left(g(x) - f(x)\right)$$

$$\text{Area} = \frac{1}{2}\pi \left(\frac{1}{2}\left(g(x) - f(x)\right)\right)^2$$

$$V = \int_0^{\frac{\pi}{2}} \left(\frac{1}{2}\pi \left(\frac{g(x) - f(x)}{2} \right)^2 \right) dx$$

1: integrand
1: limits
1: constant

$$V = \frac{\pi}{8} \int_0^{\frac{\pi}{2}} \left(e^x - \sqrt{\cos x}\right)^2 dx$$

6. ② a. $\lim_{x\to 1^-} f(x) = 7$; $\lim_{x\to 1^+} f(x) = 7$; $f(1) = 7$

considers
1: one-sided limits
1: answer, jya

By the definition of continuity, $f(x)$ is continuous at $x=1$.

④ b. Check endpoints and any relative minimum and relative maximum in $[-2,2]$.
Because this is a piecewise function, check differentiability at $x=1$.

$$f'(x) = \begin{cases} -2-2x, & x \le 1 \\ 4e^{x-1}, & x > 1 \end{cases} \qquad -2-2x=0 \quad x=-1 \qquad 4e^{x-1} \neq 0$$

At $x=1$, $f(x)$ is not differentiable because the left hand derivative $= -4$ and the right hand derivative $= 4$

1: $f'(x)$
1: identifies $x=-1$ and $x=1$
1: evaluates at endpts
1: answer, Jya

x	$f(x)$
-2	$10+4-4=10$
-1	$10+2-1=11$
1	7
2	$3+4e$

The absolute maximum is at $f(2)=3+4e$.
The absolute minimum is at $f(1)=7$.

③ c. $\int_0^2 f(x)\,dx = \int_0^1 (10-2x-x^2)\,dx + \int_1^2 (3+4e^{x-1})\,dx$

$= \left(10x - x^2 - \frac{1}{3}x^3\right)\Big|_0^1 + \left(3x+4e^{x-1}\right)\Big|_1^2$

$= 10-1-\frac{1}{3}-0+6+4e-3-4$

$= \boxed{\frac{23}{3}+4e}$

1: Sum of integrals
1: antiderivatives
1: answer