Area Appx from Wednesday 4/25

1. a. $V \approx 2(50.3) + 3(14.4) + 5(6.5) = \boxed{176.3\ ft^3}$

b. The approximation is part a is an overestimate because $A(h)$ is concave up.

c. $V = \int_0^{10} \left(\frac{50.3}{e^{0.2h} + h}\right) = \boxed{101.325\ ft^3}$

d. Find $\frac{dV}{dt}$ when $h = 5\ ft$ and $\frac{dh}{dt} = 0.26\ ft/min$

$$V = \int_0^h f(x)\,dx$$

$$\frac{dV}{dt} = \frac{d}{dt}\int_0^h f(x)\,dx = f(h)\frac{dh}{dt}$$

At $h=5$: $\frac{dV}{dt} = f(5)(0.26) = (6.517)(0.26)$
$= \boxed{1.694\ ft^3/min}$

1. a. $R'(2) \approx \dfrac{R(3) - R(1)}{3-1} = \dfrac{950-1190}{2} = \boxed{-120 \text{ liters/hr}^2}$

b. Total Amt Water Removed $\approx 1(1340)+2(1190)+3(950)+2(740)$

$= \boxed{8050 \text{ liters}}$

Because $R(t)$ is decreasing, this is an overestimate.

c. Water in tank $\approx$ 50,000 + water in − water out

$= 50{,}000 + \int_0^8 W(t)\,dt - 8050$

$= \boxed{49{,}786 \text{ liters}}$

d. $w(t) = R(t)$ $\qquad w(0) = 2000 \quad w(8) = 81.524$

$w(t) - R(t) = 0$ $\qquad R(0) = 1340 \quad R(8) = 700$

$w(0) - R(0) > 0$

$w(8) - R(8) < 0$

By the Intermediate Value Theorem, there is at least one time t, $0 < t < 8$, such that the water pumped into the tank equals the water removed from the tank.

3\. a. $C'(3.5) \approx \frac{C(4)-C(3)}{4-3} = \frac{12.8-11.2}{4-3} = \boxed{1.6 \text{ oz/min}}$

b. $\frac{C(6)-C(0)}{6-0} = \frac{29}{12} > 2.$

Because $C(t)$ is differentiable, it is continuous on the closed interval $[0,6]$, so by the MVT there is at least one time t between $t=2$ and $t=4$ that $C'(t)=2$.

c. $\frac{1}{6}\int_0^6 C(t)\,dt \approx \frac{1}{6}\left(2(5.3)+2(11.2)+2(13.8)\right)$

$= \frac{1}{6}(10.6+22.4+27.6)$

$= \frac{1}{6}(60.6)$

$= \boxed{10.1 \text{ oz}}$

$\frac{1}{6}\int_0^6 C(t)\,dt$ represents the average amount of coffee, in ounces, in the cup between $t=0$ and $t=6$ minutes.

d. $B'(t) = -16e^{-0.4t}(-0.4) = 6.4e^{-0.4t}$

$B'(5) = 6.4e^{-0.4(5)} = \boxed{\frac{6.4}{e^2} \text{ oz/min}}$

2\. a. $\frac{E(7)-E(5)}{7-5} = \frac{21-13}{7-5} = \boxed{4 \text{ hundred entries/hour}}$

b. $\frac{1}{8}\int_0^8 E(t)\,dt \approx \frac{1}{8}\left(\frac{2}{2}(0+4)+\frac{3}{2}(4+13)+\frac{2}{2}(13+21)+\frac{1}{2}(21+23)\right)$

$= \frac{1}{8}(85.5) = \boxed{10.688 \text{ hundred entries}}$

$\frac{1}{8}\int_0^8 E(t)\,dt$ represents the average number of entries in the box between 12 noon and 8pm.

c. #Entries = Initial at 8 pm − # processed

$= 23 - \int_8^{12} P(t)\,dt = 23 - 16 = \boxed{7 \text{ hundred entries}}$

d. Most quickly = Absolute maximum

$P(t)$ = rate processed. Find absolute maximum of $P(t)$

$P'(t) = 3t^2 - 60t + 298 = 0$

$t = 9.184$ Relative maximum because f' changes from positive to negative there.

$t = 10.816$ Relative minimum because f' changes from negative to positive there.

t	$P(t)$
8	0
9.184	5.089
12	8

Entries are processed most quickly at $t = 12$ midnight