Word Problems #8

1. Amt grass clippings = $A(t) = 6.687(0.931)^t$, $0 \le t \le 30$

ⓐ Average rate of change = $\dfrac{A(30) - A(0)}{30-0} = \dfrac{0.783 - 6.687}{30}$

$$= \boxed{-0.197 \text{ lbs/day}}$$

ⓑ $A'(15) = -0.164$ (use calculator)

The amount of grass clippings in the bin is decreasing by 0.164 lbs/day at $t = 15$ days.

ⓒ when is $A(t) = \frac{1}{30}\int_0^{30} A(t)\,dt$?

$$6.687(0.931)^t = 2.753$$

$$(0.931)^t = \frac{2.753}{6.687}$$

$$t = \log_{0.931}\left(\frac{2.753}{6.687}\right) = \frac{\ln\left(\frac{2.753}{6.687}\right)}{\ln 0.931}$$

$$\boxed{t = 12.413 \text{ days}}$$

ⓓ Need linear approximation to A at $t = 30$.

Find equation of tangent line to $A(t)$ at $t = 30$.

point: $t = 30$, $A(30) = 0.783$

slope: $A'(30) = -0.060$

$$y - 0.783 = -0.060(t - 30)$$

$$y = -0.060(t-30) + 0.783$$

$$0.5 = -0.060(t-30) + 0.783$$

$$t = 34.717 \text{ days}$$

If no round-off:

$$y - A(30) = A'(30)(t-30)$$

$$y = A(30) + A'(30)(t-30)$$

$$0.5 = A(30) + A'(30)(t-30)$$

$$\rightarrow t = 30 + \frac{0.5 - A(30)}{A'(30)}$$

$$t = 35.054 \text{ days}$$

2. Rate of rain <u>into</u> pipe = $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ ft^3/hr, $0<t<8$

Rate of rain <u>out of</u> pipe = $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ ft^3/hr

(0 hours, 30 ft^3)

a. Amt of water = $\int_0^8 R(t)\,dt = \boxed{76.570\ ft^3}$

b. Is $R(3) - D(3)$ positive or negative?

$5.086 - 5.4 < 0$

$\boxed{\text{Because } R(3) < D(3)\text{, the amount in the pipe is decreasing.}}$

c. Amt of water at any time $t = 30 + \int_0^t \left(R(x) - D(x)\right)dx$

Need to check endpoints and any relative minimum.

$\frac{dA}{dt} = R(t) - D(t) = 0$ when $t = 0,\ 3.272$

t	Amount
0	30
3.272	27.965
8	48.544

$\boxed{\text{The amount of water in the pipe is at a minimum at } t = 3.272 \text{ hrs.}}$

d. $50 = 30 +$ amt in $-$ amt out

$$\boxed{50 = 30 + \int_0^w R(t)\,dt - \int_0^w D(t)\,dt}$$

3. Rate of removal $= f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right)$, $0 < t \le 12$

Rate of addition $= g(t) = 3 + 2.4\ln(t^2 + 2t)$ for $3 \underline{\le} t < 12$

Point (0 hours, 50 bananas)

ⓐ Number pounds removed $= \int_0^{t} f(t)\,dt$

Number pounds removed during first 2 hours:

$= \int_0^2 f(t)\,dt = \boxed{20.051 \text{ lbs}}$

ⓑ $f'(7) = -8.120$ lbs (use calculator)

The number of pounds of bananas on the display table is decreasing by 8.120 lbs/hour 7 hours after the store opens.

ⓒ Is $g(t) - f(t)$ positive or negative at $t = 5$?

$g(5) - f(5)$

$11.533 - 13.796 < 0$ therefore, the number of pounds of bananas on the display table is decreasing.

OR

$A(t) =$ Amt bananas on table $= 50 + \int_0^t (g(x) - f(x))\,dx$

$A'(t) = g(t) - f(t)$

$A'(5) = g(5) - f(5) < 0$

ⓓ At $t = 8$:

$50 - \int_0^8 f(t)\,dt + \int_3^8 g(t)\,dt = \boxed{23.347 \text{ lbs bananas}}$