#7 Area and Volume Part B

1. No Calc

radius $= r = \frac{1}{20}(3+h^2)$ in $\quad 0 \le h \le 10 \quad$ h = height inches

a. $\frac{1}{10-0}\int_0^{10} \frac{1}{20}(3+h^2)\,dh = \frac{1}{200}\left(3h + \frac{1}{3}h^3\right)\Big|_0^{10}$

$= \frac{1}{200}\left(30 + \frac{1000}{3} - 0\right) = \frac{1090}{600} = \boxed{\frac{109}{60} \text{ in}}$

b. h = 10 inches

Circular cross sections $\quad$ Area $= \pi r^2$

$= \pi\left(\frac{1}{20}(3+h^2)\right)^2$

$V = \int_0^{10} \overset{\text{Formula}}{\left(\text{Area of one cross Section}\right)}\,dh$

$V = \int_0^{10} \pi\left(\frac{1}{20}(3+h^2)\right)^2 = \frac{\pi}{400}\int_0^{10}(3+h^2)(3+h^2)\,dh$

$= \frac{\pi}{400}\int_0^{10}(9+6h^2+h^4)\,dh = \frac{\pi}{400}\left(9h + 2h^3 + \frac{1}{5}h^5\right)\Big|_0^{10}$

$= \frac{\pi}{400}\left(90 + 2000 + \frac{100{,}000}{5}\right) = \frac{\pi}{400}(22090) = \boxed{\frac{2209\pi}{400} \text{ in}^3}$

c. Find $\frac{dh}{dt}$ when $h = 3$ in, $\frac{dr}{dt} = -\frac{1}{5}$ in/sec

$r = \frac{1}{20}(3+h^2)$

$\frac{dr}{dt} = \frac{1}{20}(2h)\frac{dh}{dt}$

$-\frac{1}{5} = \frac{1}{20}(6)\frac{dh}{dt}$

$\frac{dh}{dt} = \left(-\frac{1}{5}\right)\left(\frac{20}{6}\right) = \boxed{-\frac{2}{3} \text{ in/sec}}$

calculator

2. $f(x) = 1 + x + e^{x^2-2x}$ $\qquad$ $g(x) = x^4 - 6.5x^2 + 6x + 2$

ⓐ $f(x)$ and $g(x)$ intersect:

$f(x) = g(x)$ graph both, find intersection

point $(1.033, 2.401)$

$$\text{Area of } R+S = \int_0^{1.033} (g(x) - f(x))\,dx + \int_{1.033}^{2} (f(x) - g(x))\,dx$$

$$= 0.997 + 1.007 = \boxed{2.004}$$

ⓑ Area Square $= (f(x) - g(x))^2$

$$V = \int_{1.033}^{2} (f(x) - g(x))^2\,dx = 1.283$$

ⓒ $h = f(x) - g(x)$ the Vertical distance between f and g in region S.

Find $\frac{dh}{dt}$ when $x = 1.8$

$h = f(x) - g(x)$

$\frac{dh}{dt} = f'(x) - g'(x)$

when $x = 1.8$:

$\frac{dh}{dt} = f'(1.8) - g'(1.8)$. Use calculator to find derivatives.

$= 2.116 - 5.928$

$= \boxed{-3.812}$

calculator

3. $y = \ln x \quad y = 5 - x$

ⓐ $y = \ln x$ intersects $y = 5 - x$ at $(3.693, 1.307)$

$$\text{Area} = \int_1^{3.693} (\ln x - 0)\,dx + \int_{3.693}^{5} (5 - x - 0)\,dx$$

$$= 2.132 + (0.854) = \boxed{2.986}$$

ⓑ Area of square $= (\text{side})^2$

$$\boxed{V = \int_0^{3.693} (\ln x)^2\,dx + \int_{3.693}^{5} (5-x)^2\,dx}$$

ⓒ Horizontal line $y = k$.

The integral will be dy!

$y = \ln x \qquad y = 5 - x$

$x = e^y \qquad x = 5 - y$

$$\int_0^k (5 - y - e^y)\,dy = \int_k^{1.307} (5 - y - e^y)\,dy$$

OR

$$\int_0^k (5 - y - e^y)\,dy = \frac{1}{2}(2.986)$$

from answer to part a.

No calc!

4. $f(x) = 8x^3$ $\qquad g(x) = \sin(\pi x)$

ⓐ Eqn. of tangent line to f at $x = \frac{1}{2}$.

Point: $x = \frac{1}{2}$, $f(\frac{1}{2}) = 1$

slope: $f'(x) = 24x^2 \Big|_{x=\frac{1}{2}} = 6$

$$\boxed{y - 1 = 6\left(x - \tfrac{1}{2}\right)}$$

ⓑ Area $R = \int_0^{\frac{1}{2}} (g(x) - f(x))\,dx$

$= \int_0^{1/2} (\sin(\pi x) - 8x^3)\,dx = \left(-\frac{1}{\pi}\cos(\pi x) - 2x^4\right)\Big|_0^{1/2}$

$= \left(-\frac{1}{\pi}(0) - \frac{1}{8}\right) - \left(-\frac{1}{\pi} - 0\right) = \boxed{-\frac{1}{8} + \frac{1}{\pi}}$

ⓒ $R = 1 - f(x)$

$r = 1 - g(x)$

$$\boxed{V = \pi \int_0^{\frac{1}{2}} \left((1 - f(x))^2 - (1 - g(x))^2\right) dx}$$