Evaluate each limit.

1) $\lim_{x \to -\infty} \frac{3x^3}{2x^2 + 4}$

2) $\lim_{x \to \infty} \frac{3x^2}{x^2 - 9}$

3) $\lim_{x \to -2^-} f(x),\ f(x) = \begin{cases} -x^2 - 8x - 16, & x < -2 \\ x + 2, & x \ge -2 \end{cases}$

4) $\lim_{x \to 1^+} f(x),\ f(x) = \begin{cases} -x^2 + 2x - 1, & x \le 1 \\ x - 7, & x > 1 \end{cases}$

5) $\lim_{x \to 1} f(x),\ f(x) = \begin{cases} -1 + \frac{x}{2}, & x \ne 1 \\ -5, & x = 1 \end{cases}$

6) $\lim_{x \to -2} f(x),\ f(x) = \begin{cases} 2 + \frac{x}{2}, & x \ne -2 \\ -1, & x = -2 \end{cases}$

7) $\lim_{x \to 0} \frac{e^{5x} - 1}{4x}$

8) $\lim_{x \to 0} \frac{2x}{\ln (x + 1)}$

9) $\lim_{x \to 0} \frac{4x}{\ln (x + 1)}$

10) $\lim_{x \to 0} \frac{3(e^x - e^{-x})}{\sin x}$

For each problem, find the values of *c* that satisfy the Mean Value Theorem.

11) $y = -2x^2 + 12x - 15;\ [2, 5]$

12) $y = -2x^2 - 4x + 3;\ [-2, 1]$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \ge 0$. For each problem, find the intervals of time when the particle is slowing down and speeding up.

13) $s(t) = -t^3 + 15t^2$

14) $s(t) = -t^3 + 10t^2$

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

15) $y = -\cot (x)$ at $\left(-\frac{3\pi}{4}, -1\right)$

16) $y = \tan (2x)$ at $\left(-\frac{\pi}{6}, -\sqrt{3}\right)$

For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

17)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	2	1	2
2	4	$\frac{1}{2}$	3	$\frac{3}{2}$
3	3	-1	4	$-\frac{1}{2}$
4	2	-1	2	-2

Given $h(x) = f(g(x))$, find $h'(3)$

18)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	2	4	-2
2	4	$\frac{1}{2}$	2	$-\frac{3}{2}$
3	3	-1	1	0
4	2	-1	2	1

Given $h(x) = f(g(x))$, find $h'(2)$

Differentiate each function with respect to *x*.

19) $y = \sec x^5$

20) $y = \sec x^4$

21) $y = \ln 3x^2$

22) $y = e^{x^5}$

23) $y = e^{3x^3}$

24) $y = \ln 4x^4$

25) $y = \frac{3x^4}{x^5 + 4}$

26) $y = \frac{3x^3}{2x^4 + 4}$

For each problem, find all points of relative minima and maxima.

27) $y = x^3 - 12x^2 + 45x - 56$

28) $y = x^3 - 12x^2 + 45x - 52$

For each problem, determine if Rolle's Theorem can be applied. If it can, find all values of *c* that satisfy the theorem. If it cannot, explain why not.

29) $y = \cos(2x); \quad [-\pi, \pi]$

30) $y = 2\sin(2x); \quad [-\pi, \pi]$

For each problem, find the open intervals where the function is concave up and concave down.

31) $y = x^5 - 2x^3 + 1$

32) $y = -x^4 + x^2 + 1$

Answers to

1) $-\infty$

2) 3

3) -4

4) -6

5) $-\frac{1}{2}$

6) 1

7) $\frac{5}{4}$

8) 2

9) 4

10) 6

11) $\left\{\frac{7}{2}\right\}$

12) $\left\{-\frac{1}{2}\right\}$

13) Slowing down: $5 < t < 10$, Speeding up: $0 < t < 5$, $t > 10$

14) Slowing down: $\frac{10}{3} < t < \frac{20}{3}$, Speeding up: $0 < t < \frac{10}{3}$, $t > \frac{20}{3}$

15) $y = 2x + \frac{-2 + 3\pi}{2}$

16) $y = 8x + \frac{-3\sqrt{3} + 4\pi}{3}$

17) $h'(3) = \frac{1}{2}$

18) $h'(2) = -\frac{3}{4}$

19) $\frac{dy}{dx} = \sec x^5 \tan x^5 \cdot 5x^4$
$= 5x^4 \sec x^5 \tan x^5$

20) $\frac{dy}{dx} = \sec x^4 \tan x^4 \cdot 4x^3$
$= 4x^3 \sec x^4 \tan x^4$

21) $\frac{dy}{dx} = \frac{1}{3x^2} \cdot 6x$
$= \frac{2}{x}$

22) $\frac{dy}{dx} = e^{x^5} \cdot 5x^4$

23) $\frac{dy}{dx} = e^{3x^3} \cdot 9x^2$

24) $\frac{dy}{dx} = \frac{1}{4x^4} \cdot 16x^3$
$= \frac{4}{x}$

25) $\frac{dy}{dx} = \frac{(x^5 + 4) \cdot 12x^3 - 3x^4 \cdot 5x^4}{(x^5 + 4)^2}$
$= \frac{-3x^8 + 48x^3}{x^{10} + 8x^5 + 16}$

26) $\frac{dy}{dx} = \frac{(2x^4 + 4) \cdot 9x^2 - 3x^3 \cdot 8x^3}{(2x^4 + 4)^2}$
$= \frac{-3x^6 + 18x^2}{2x^8 + 8x^4 + 8}$

27) Relative minimum: $(5, -6)$
Relative maximum: $(3, -2)$

28) Relative minimum: $(5, -2)$
Relative maximum: $(3, 2)$

29) $\left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

30) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}\right\}$

31) Concave up: $\left(-\frac{\sqrt{15}}{5}, 0\right), \left(\frac{\sqrt{15}}{5}, \infty\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{15}}{5}\right), \left(0, \frac{\sqrt{15}}{5}\right)$

32) Concave up: $\left(-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{6}}{6}\right), \left(\frac{\sqrt{6}}{6}, \infty\right)$