Differential Equations: Find the particular solution to the following differential equations.

1. $\frac{dy}{dx} = (3-y)\cos x\,;\, y(0) = 1$

2. $\frac{dy}{dx} = e^{y}(3x^2 - 6x)\,;\,(1,0)$

3. $\frac{dB}{dx} = \frac{1}{5}(100 - B)\,;\, B(0) = 20$

4. $\frac{dW}{dx} = \frac{1}{25}(W - 300)\,;\, W(0) = 1400$

5. $\frac{dy}{dx} = xy^3\,;\, f(1) = 2$

6. $\frac{dy}{dx} = \frac{y-1}{x^2}\,;\, f(2) = 0$

7. $\frac{dy}{dx} = \frac{1+y}{x}\,;\, f(-1) = 1$

8. $\frac{dy}{dx} = (y-1)^2\cos(\pi x); f(1) = 0$

9. $\frac{dy}{dx} = \frac{-2x}{y}\,;\, f(1) = -1$

10. $\frac{dy}{dx} = \frac{-xy^2}{2}\,;\, f(-1) = 2$

Warm Up #8

1. $\frac{dy}{dx} = (3-y)\cos x \qquad y(0)=1$

$\frac{1}{3-y}\,dy = \cos x\,dx$

$-\ln|3-y| = \sin x + c$

$y(0)=1$

$-\ln|3-1| = \sin 0 + c$

$c = -\ln 2$

$-\ln|3-y| = \sin x - \ln 2$

(Because $y(0)=1$, $y<3$
so $|3-y| = 3-y$)

$\ln(3-y) = -\sin x + \ln 2$

$e^{\ln(3-y)} = e^{-\sin x + \ln 2}$

$3-y = 2e^{-\sin x}$

$\boxed{y = 3-2e^{-\sin x}}$

2. $\frac{dy}{dx} = e^{y}(3x^2-6x) \qquad (1,0)$

$e^{-y}\,dy = (3x^2-6x)\,dx$

$-e^{-y} = x^3-3x^2+c$

$(1,0)$

$-1 = 1-3+c$

$c = 1$

$-e^{-y} = x^3-3x^2+1$

$e^{-y} = -x^3+3x^2-1$

$\ln e^{-y} = \ln(-x^3+3x-1)$

$-y = \ln(x^3+3x-1)$

$\boxed{y = -\ln(x^3+3x-1)}$

3. $\frac{dB}{dt} = \frac{1}{5}(100-B) \qquad B(0)=20$

$\frac{1}{100-B}\,dB = \frac{1}{5}\,dt$

$-\ln|100-B| = \frac{1}{5}t + c$

$B(0)=20$

$-\ln 80 = c$

$-\ln|100-B| = \frac{1}{5}t - \ln 80$

(Because $B(0)=20$, $B<100$
so $|100-B| = 100-B$)

$\ln(100-B) = -\frac{1}{5}t + \ln 80$

$e^{\ln(100-B)} = e^{-\frac{1}{5}t + \ln 80}$

$100-B = 80e^{-\frac{1}{5}t}$

$\boxed{B = 100-80e^{-\frac{1}{5}t}}$

4. $\frac{dW}{dt} = \frac{1}{25}(W-300)$ $\quad\quad$ $W(0) = 1400$

$\frac{1}{W-300}\, dW = \frac{1}{25}\, dt$

$\ln|W-300| = \frac{1}{25}t + C$ $\longrightarrow$ $\ln|W-300| = \frac{1}{25}t + \ln 1100$

$W(0) = 1400$ $\quad\quad$ (Because $W(0)=1400$, $W < 1400$

$\ln 1100 = C$ $\quad\quad$ so $|W-300| = W-300$)

$e^{\ln(W-300)} = e^{\frac{1}{25}t + \ln 1100}$

$W - 300 = 1100e^{\frac{1}{25}t}$

$\boxed{W = 300 + 1100e^{\frac{1}{25}t}}$

5. $\frac{dy}{dx} = xy^3$ $\quad\quad$ $f(1) = 2$

$\frac{1}{y^3}\, dy = x\, dx$

$\frac{-1}{2y^2} = \frac{1}{2}x^2 + C$ $\longrightarrow$ $-\frac{1}{2y^2} = \frac{x^2}{2} - \frac{5}{8} = \frac{4x^2-5}{8}$

$f(1) = 2$ $\quad\quad$ $-2y^2 = \frac{8}{4x^2-5}$

$-\frac{1}{8} = \frac{1}{2} + C$ $\quad\quad$ $y^2 = \frac{-4}{4x^2-5} = \frac{4}{5-4x^2}$

$C = -\frac{5}{8}$ $\quad\quad$ $\boxed{y = \frac{2}{\sqrt{5-4x^2}}}$

6. $\frac{dy}{dx} = \frac{y-1}{x^2}$ $\quad\quad$ $f(2) = 0$

$\frac{1}{y-1}\, dy = \frac{1}{x^2}\, dx$

$\ln|y-1| = -\frac{1}{x} + C$

$|y-1| = e^{-\frac{1}{x} + C}$

$y - 1 = Ce^{-\frac{1}{x}}$ $\longrightarrow$ $y - 1 = -e^{\frac{1}{2}} e^{-\frac{1}{x}}$

$f(2) = 0$ $\quad\quad$ $\boxed{y = 1 - e^{(\frac{1}{2} - \frac{1}{x})}}$

$-1 = Ce^{-\frac{1}{2}}$

$C = \frac{-1}{e^{-\frac{1}{2}}} = -e^{\frac{1}{2}}$

7. $\frac{dy}{dx} = \frac{1+y}{x}$ $\quad f(-1)=1$

$\frac{1}{1+y}\,dy = \frac{1}{x}\,dx$

$\ln|1+y| = \ln|x| + c$

$e^{\ln|1+y|} = e^{\ln|x| + c}$

$|1+y| = C|x|$

$f(-1)=1$

$2 = C$

$|1+y| = 2|x|$

Because $f(-1)=1$, $1+y>0$ so,

$1+y = 2|x|$

$\boxed{y = 2|x| + 1}$

8. $\frac{dy}{dx} = (y-1)^2\cos(\pi x)$ $\quad f(1)=0$

$(y-1)^{-2}\,dy = \cos(\pi x)\,dx$

$\frac{-1}{y-1} = \frac{1}{\pi}\sin(\pi x) + c$

$f(1)=0$

$1 = \frac{1}{\pi}\sin\pi + c$

$c = 1$

$\frac{-1}{y-1} = \frac{1}{\pi}\sin(\pi x) + 1$

$-\frac{1}{y-1} = \frac{\sin(\pi x) + \pi}{\pi}$

$-(y-1) = \frac{\pi}{\sin(\pi x) + \pi}$

$y-1 = \frac{-\pi}{\sin(\pi x) + \pi}$

$\boxed{y = 1 - \frac{\pi}{\sin(\pi x) + \pi}}$

9. $\frac{dy}{dx} = -\frac{2x}{y}$ $\quad f(1)=-1$

$y\,dy = -2x\,dx$

$\frac{1}{2}y^2 = -x^2 + c$

$f(1)=-1$

$\frac{1}{2} = -1 + c$

$c = \frac{3}{2}$

$\frac{1}{2}y^2 = -x^2 + \frac{3}{2}$

$y^2 = -2x^2 + 3$

$y = \pm\sqrt{-2x^2+3}$

$\boxed{y = -\sqrt{-2x^2+3}}$

Because

$f(1) = -1$ negative!

10. $\frac{dy}{dx} = -\frac{xy^2}{2}$ $\qquad f(-1) = 2$

$\frac{1}{y^2}\,dy = -\frac{1}{2}x\,dx$

$-\frac{1}{y} = -\frac{1}{4}x^2 + C$

$f(-1) = 2$

$-\frac{1}{2} = -\frac{1}{4} + C$

$C = -\frac{1}{4}$

$\longrightarrow \; -\frac{1}{y} = -\frac{1}{4}x^2 - \frac{1}{4}$

$-\frac{1}{y} = \frac{-x^2-1}{4}$

$-y = \frac{4}{-x^2-1}$

$\boxed{y = \frac{4}{x^2+1}}$