DEQ Practice

find the general solution to the following differential equations, then find the particular solution using the initial condition.

4. $\frac{dy}{dx}=\frac{x}{y},\ y(1)=2$

5. $\frac{dy}{dx}=-\frac{x}{y},\ y(4)=3$

6. $\frac{dy}{dx}=\frac{y}{x},\ y(2)=2$

7. $\frac{dy}{dx}=2xy,\ y(0)=3$

8. $\frac{dy}{dx}=(y+5)(x+2),\ y(0)=-1$

9. $\frac{dy}{dx}=\cos^2 y,\ y(0)=0$

10. $\frac{dy}{dx}=(\cos x)e^{y+\sin x},\ y(0)=0$

11. $\frac{dy}{dx}=e^{x-y},\ y(0)=2$

12. $\frac{dy}{dx}=-2xy^2,\ y(1)=0.25$

13. $\frac{dy}{dx}=\frac{4\sqrt{y}\ln x}{x},\ y(e)=1$

14. For each of #4 - #13, write an equation for the line tangent to the graph of y at the given point.

1. $y\,dy = x\,dx$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + c$$
$$y^2 = x^2 + c$$
$$(1,2):\ 4 = 1 + c$$
$$c = 3$$
$$y^2 = x^2 + 3$$
$$\boxed{y = \sqrt{x^2+3}}$$

2. $\frac{1}{y}\,dy = 2x\,dx$

$$\ln|y| = x^2 + c$$
$$(0,3):\ \ln 3 = 0 + c$$
$$c = \ln 3$$
$$\ln|y| = x^2 + \ln 3$$
$$y = e^{x^2+\ln 3} = e^{x^2} e^{\ln 3}$$
$$\boxed{y = 3e^{x^2}}$$

3. $\frac{dy}{dx} = (\cos x)(e^y)(e^{\sin x})$

$$e^{-y}\,dy = \cos x\, e^{\sin x} \quad \rightarrow u = \sin x,\ du = \cos x$$
$$-e^{-y} = \int e^u\,du$$
$$-e^{-y} = e^u + c \quad \longrightarrow \text{Resubstitute } u = \sin x$$
$$-e^{-y} = e^{\sin x} + c$$
$$e^{-y} = -e^{\sin x} + c$$
$$-y = \ln(-e^{\sin x} + c)$$
$$y = -\ln(-e^{\sin x} + c)$$
$$(0,0):\ 0 = -\ln(-e^0 + c)$$
$$0 = -\ln(-1 + c)$$
$$0 = +\ln(-1 + c)$$
$$e^0 = e^{\ln(-1+c)}$$
$$1 = -1 + c$$
$$c = 2$$

$$\boxed{y = -\ln(-e^{\sin x} + 2)}$$

4. $y^{-\frac{1}{2}}\,dy = 4\left(\frac{1}{x}\right)\ln x\,dx \qquad \left(u=\ln x\,;\; du=\frac{1}{x}\,dx\right)$

$2\sqrt{y} = 4\left(\frac{1}{2}\right)(\ln x)^2 + c$

$\sqrt{y} = (\ln x)^2 + c$

$(e,1):\quad 1 = (\ln e)^2 + c$

$1 = 1 + c$

$c = 0$

$\sqrt{y} = (\ln x)^2$

$\boxed{y = (\ln x)^4}$

5. $y\,dy = -x\,dx \qquad \frac{1}{2}y^2 = -\frac{1}{2}x^2 + c$

$y^2 = -x^2 + c$

$(4,3):\quad 9 = -16 + c$

$c = 25$

$y^2 = -x^2 + 25$

$\boxed{y = \sqrt{-x^2+25}}$

6. $\frac{1}{y+5}\,dy = (x+2)\,dx \qquad \ln|y+5| = \frac{1}{2}x^2 + 2x + c$

$(0,-1):\ \ln 4 = c$

$\ln|y+5| = \frac{1}{2}x^2 + 2x + \ln 4$

$|y+5| = e^{\frac{1}{2}x^2 + 2x + \ln 4}$

$y+5 = 4e^{\frac{1}{2}x^2+2x}$

$\boxed{y = 4e^{\frac{1}{2}x^2+2x} - 5}$

7. $\frac{dy}{dx} = \frac{e^x}{e^y} \qquad e^y\,dy = e^x\,dx$

$e^y = e^x + c$

$(0,2):\quad e^2 = e^0 + c$

$c = e^2 - 1$

$e^y = e^x + e^2 - 1$

$\boxed{y = \ln(e^x + e^2 - 1)}$

8. $\frac{1}{y}\,dy = \frac{1}{x}\,dx$

$\ln|y| = \ln|x| + c$

$(2,2):\ \ln 2 = \ln 2 + c$

$c = 0$

$\ln|y| = \ln|x|$

$\boxed{y = x}$

9. $\frac{1}{\cos^2 y}\,dy = dx$

$\sec^2 y\,dy = dx$

$\tan y = x + c$

$(0,0):\ \tan 0 = c$

$c = 0$

$\tan y = x$

$\boxed{y = \tan^{-1} x}$

10. $\frac{1}{y^2}\,dy = -2x\,dx$

$-\frac{1}{y} = -x^2 + c$

$(1, 0.25)$

$(1, \frac{1}{4}):\ -\frac{1}{\frac{1}{4}} = -1 + c$

$-4 = -1 + c$

$c = -3$

$-\frac{1}{y} = -x^2 - 3$

$\frac{1}{y} = x^2 + 3$

$\boxed{y = \frac{1}{x^2+3}}$

#11: 1) $\frac{dy}{dx}$ at $(1,2) = \frac{1}{2}$ $\boxed{y-2=\frac{1}{2}(x-1)}$

2) $\frac{dy}{dx}$ at $(0,3) = 0$ $\boxed{y-3=0}$ $y=3$

3) $\frac{dy}{dx}$ at $(0,0) = 1$ $\boxed{y-0=1(x-0)}$ $y=x$

4) $\frac{dy}{dx}$ at $(e,1) = \frac{4\sqrt{e}\ln 1}{1}$ $\boxed{y-1=0(x-e)}$ $y=1$

$= 0$

5) $\frac{dy}{dx}$ at $(4,3) = -\frac{4}{3}$ $\boxed{y-3=-\frac{4}{3}(x-4)}$

6) $\frac{dy}{dx}$ at $(0,-1) = (-1+5)(0+2)$ $\boxed{y+1=8(x-0)}$

$= 8$

7) $\frac{dy}{dx}$ at $(0,2) = e^{0-2} = \frac{1}{e^2}$ $\boxed{y-2=\frac{1}{e^2}(x-0)}$

8) $\frac{dy}{dx}$ at $(2,2) = \frac{2}{2} = 1$ $\boxed{y-2=1(x-2)}$

9) $\frac{dy}{dx}$ at $(0,0) = \cos^2 0 = 1$ $\boxed{y-0=1(x-0)}$ $y=x$

10) $\frac{dy}{dx}$ at $(1,\frac{1}{4}) = -2(1)(\frac{1}{4})^2 = -\frac{1}{8}$ $\boxed{y-\frac{1}{4}=-\frac{1}{8}(x-1)}$