Classwork - Friday Jan. 5, 2018

3. $\int 4(6x-1)^{2/3}\,dx$

$u = 6x-1$
$du = 6dx$

$= \left(\frac{1}{6}\right)4\int 6(6x-1)^{2/3}\,dx = \frac{2}{3}\int u^{\frac{2}{3}}\,du = \frac{2}{3}\left(\frac{u^{\frac{5}{3}}}{\frac{5}{3}}\right)+c$

$= \left(\frac{3}{5}\right)\left(\frac{2}{3}\right)u^{\frac{5}{3}}+c = \boxed{\frac{2}{5}(6x-1)^{5/3}+c}$

4. $\int x\sqrt{x^2-2}\,dx = \int x(x^2-2)^{\frac{1}{2}}\,dx$

$u = x^2-2$
$du = 2x\,dx$

$= \frac{1}{2}\int 2x(x^2-2)^{\frac{1}{2}}\,dx = \frac{1}{2}\int u^{\frac{1}{2}}\,du$

$= \frac{1}{2}\left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right)+c = \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)u^{\frac{3}{2}}+c$

$= \frac{1}{3}u^{\frac{3}{2}}+c = \boxed{\frac{1}{3}(x^2-2)^{\frac{3}{2}}+c}$

5. $\int x^2\sqrt{1-4x^3}\,dx = \int x^2(1-4x^3)^{\frac{1}{2}}\,dx$

$u = 1-4x^3$
$du = -12x^2\,dx$

$= \frac{1}{-12}\int -12x^2(1-4x^3)^{\frac{1}{2}}\,dx$

$= -\frac{1}{12}\int u^{\frac{1}{2}}\,du = \left(-\frac{1}{12}\right)\left(\frac{2}{3}u^{\frac{3}{2}}\right)+c$

$= \boxed{-\frac{1}{18}(1-4x^3)^{\frac{3}{2}}+c}$

6. $\int \frac{x}{\sqrt[3]{2x^2-1}}\,dx = \int x(2x^2-1)^{-\frac{1}{3}}\,dx$

$u = 2x^2-1$
$du = 4x\,dx$

$= \frac{1}{4}\int 4x(2x^2-1)^{-\frac{1}{3}}\,dx$

$= \frac{1}{4}\int u^{-\frac{1}{3}}\,du = \left(\frac{1}{4}\right)\left(\frac{3}{2}\right)u^{\frac{2}{3}}+c$

$= \boxed{\frac{3}{8}(2x^2-1)^{\frac{2}{3}}+c}$

7. $\int x^{\frac{1}{2}}\left(x^{\frac{3}{2}}+4\right)^{9}dx = \frac{2}{3}\int \frac{3}{2}x^{\frac{1}{2}}\left(x^{\frac{3}{2}}+4\right)^{9}dx = \frac{2}{3}\int u^{9}\,du$

$u = x^{\frac{3}{2}}+4$

$du = \frac{3}{2}x^{\frac{1}{2}}dx$

$= \frac{2}{3}\left(\frac{1}{10}\right)u^{10}+C = \boxed{\frac{1}{15}\left(x^{\frac{3}{2}}+4\right)^{10}+C}$

8. $\int (x+2)\sqrt{x^2+4x-5}\,dx = \int (x+2)(x^2+4x-5)^{\frac{1}{2}}dx$

$u = x^2+4x-5$

$du = (2x+4)dx$

$= \frac{1}{2}\int 2(x+2)(x^2+4x-5)^{\frac{1}{2}}dx$

$= \frac{1}{2}\int (2x+4)(x^2+4x-5)^{\frac{1}{2}}dx = \frac{1}{2}\int u^{\frac{1}{2}}du$

$= \frac{1}{2}\left(\frac{2}{3}\right)u^{\frac{3}{2}}+C = \boxed{\frac{1}{3}(x^2+4x-5)^{\frac{3}{2}}+C}$

9. $\int (x-\sqrt{3x})\,dx = \int x\,dx - \int\sqrt{3x}\,dx = \int x\,dx - \int (3x)^{\frac{1}{2}}dx$

$= \frac{1}{2}x^2 - \frac{1}{3}\int 3(3x)^{\frac{1}{2}}dx = \frac{1}{2}x^2 - \frac{1}{3}\int u^{\frac{1}{2}}du$

$u = 3x$

$du = 3dx$

$= \frac{1}{2}x^2 - \frac{1}{3}\left(\frac{2}{3}\right)u^{\frac{3}{2}}+C$

$= \boxed{\frac{1}{2}x^2 - \frac{2}{9}(3x)^{\frac{3}{2}}+C}$

10. $\int\sqrt{x^2-1}\,dx = \int (x^2-1)^{\frac{1}{2}}dx$

$u = x^2-1$

$du = 2x\,dx$

Cannot use this method to find the integral!

You can <u>only</u> multiply by the constant 1.

You <u>cannot</u> multiply by a variable!

11. $\int \cos 4x\,dx = \frac{1}{4}\int 4\cos 4x\,dx = \frac{1}{4}\int \cos u\,du = \frac{1}{4}\sin u + c$

$u = 4x$

$du = 4dx$

$= \boxed{\frac{1}{4}\sin 4x + c}$

12. $\int 3\sin(1-3x)\,dx = -\int -3\sin(1-3x)\,dx = -\int \sin u \cdot du$

$u = 1-3x$

$du = -3dx$

$= \cos u + c = \boxed{\cos(1-3x) + c}$

13. $\int \sin^3 x \cos x\,dx = \int (\sin x)^3 \cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$= \int u^3\,du = \frac{1}{4}u^4 + c$

$\boxed{= \frac{1}{4}(\sin x)^4 + c = \frac{1}{4}\sin^4 x + c}$

14. $\int \cos x\sqrt{1-\sin x}\,dx = \int \cos x\,(1-\sin x)^{\frac{1}{2}}\,dx$

$u = 1-\sin x$

$du = -\cos x\,dx$

$= -\int -\cos x\,(1-\sin x)^{\frac{1}{2}}\,dx$

$= -\int u^{\frac{1}{2}}\,du = -\left(\frac{2}{3}\right)u^{\frac{3}{2}} + c$

$= \boxed{-\frac{2}{3}(1-\sin x)^{\frac{3}{2}} + c}$

15. $\int \tan 10x \sec 10x \, dx = \frac{1}{10} \int 10 \tan 10x \sec 10x \, dx$

$u = 10x$

$du = 10 dx$

$= \frac{1}{10} \int \tan u \sec u \, du = \frac{1}{10} \int \sec u \tan u \, du$

$= \frac{1}{10} \sec u + c = \boxed{\frac{1}{10} \sec 10x + c}$

16. $\int \tan^2 x \sec^2 x \, dx = \int (\tan x)^2 \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$= \int u^2 \, du = \frac{1}{3} u^3 + c$

$= \boxed{\frac{1}{3} \tan^3 x + c}$

17. $\int (x+2)\sqrt{x-4} \, dx = \int (x+2)(x-4)^{1/2} dx$

$u = x - 4 \rightarrow x = u + 4$

$du = dx$

$\int (u+4+2) u^{1/2} du$

$= \int (u^{3/2} + 6u^{1/2}) du = \frac{2}{5} u^{5/2} + (6)(\frac{2}{3}) u^{\frac{3}{2}} + c$

$= \boxed{\frac{2}{5}(x-4)^{\frac{5}{4}} + 4(x-4)^{\frac{3}{2}} + c}$

18. $\int \frac{x^2}{\sqrt{x-1}} dx = \int x^2 (x-1)^{-\frac{1}{2}} dx$

$u = x - 1 \rightarrow x = u + 1$

$du = dx$

$= \int (u+1)^2 u^{-\frac{1}{2}} du$

$= \int (u^2 + 2u + 1) u^{-\frac{1}{2}} du$

$= \int (u^{\frac{3}{2}} + 2u^{\frac{1}{2}} + u^{-\frac{1}{2}}) du$

$= \frac{2}{5} u^{\frac{5}{2}} + 2(\frac{2}{3}) u^{\frac{3}{2}} + 2u^{\frac{1}{2}} + c$

$= \boxed{\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{4}{3}(x-1)^{3/2} + 2(x-1)^{1/2} + c}$