Calculus Additional Benchmark 1 Review Name________________________

Evaluate each limit.

1) $\lim_{x \to -3} \frac{x+3}{x^2+x+1}$

2) $\lim_{x \to 1} \frac{25x}{x^2+25}$

3) $\lim_{x \to -\infty} -\frac{4x}{x^2+4}$

4) $\lim_{x \to \infty} \frac{x^2}{x^2+4}$

5) $\lim_{x \to \infty} \frac{x^4}{x^2-1}$

6) $\lim_{x \to \infty} \left(-x^3+x^2-3\right)$

7) $\lim_{x \to 3} \frac{x^2-7x+12}{x-3}$

8) $\lim_{x \to -2} -\frac{x+2}{x^2+3x+2}$

9) $\lim_{x \to 1} \frac{x^2-3x+2}{x-1}$

10) $\lim_{x \to 2} \frac{x^2+x-6}{x-2}$

11) $\lim_{x \to -2} f(x),\ f(x) = \begin{cases} -x^2+1, & x \neq -2 \\ -4, & x = -2 \end{cases}$

12) $\lim_{x \to 1} f(x),\ f(x) = \begin{cases} 1, & x \neq 1 \\ 3, & x = 1 \end{cases}$

Differentiate each function with respect to *x*.

13) $y = 2\sqrt[5]{x^2} + 2\sqrt[5]{x} - \frac{5}{3}x^{-5}$

14) $y = -2x^2 - \frac{4}{3x^3} + \frac{5}{4x^4}$

15) $y = 5x^4 - \frac{5}{3}x^3 - x^{\frac{4}{3}}$

16) $y = \left(3x^3+2\right) \cdot -2x^5$

17) $y = \left(2x^5+3\right)\left(3x^4-3\right)$

18) $y = \left(-5\sqrt[3]{x}-2\right)\left(-4x^3-5\right)$

19) $y = \frac{2x^4+2x^2}{3x^4-5}$

20) $y = \frac{2x^4+5x^3}{4x^4-5}$

21) $y = \left(2x^3-1\right)^5$

22) $y = \left(-2x^3+5\right)^{\frac{1}{3}}$

23) $y = (3x^5 + 1)\sqrt{-x^2 + 2}$

For each problem, find the average rate of change of the function over the given interval.

24) $y = -2x^2 + x - 1;\ \ [0, 2]$

25) $y = x^2 + 2;\ \ [-1, 1]$

For each problem, find the instantaneous rate of change of the function at the given value.

26) $f(x) = 2x^2 + x + 1;\ \ -2$

27) $f(x) = -\frac{1}{x + 2};\ \ -1$

For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.

28) $y = -(x - 1)^{\frac{2}{3}}$ at $(2, -1)$

29) $y = x^3 - 2x^2 + 2$ at $(-1, -1)$

For each problem, find the points where the tangent line to the function is horizontal.

30) $y = -x^2 + 4x - 5$

31) $y = \frac{2}{x^2 - 4}$

32) $y = -x^3 + 3x^2 + 2$

33) $y = -\frac{x^2}{2x - 3}$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity and acceleration at the given value for *t*.

34) $s(t) = t^3 - 12t^2;$ at $t = 8$

35) $s(t) = t^3 - 4t^2 - 60t;$ at $t = 2$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the times *t* when the particle changes directions.

36) $s(t) = -t^3 + 12t^2$

37) $s(t) = t^3 - 11t^2 + 24t$

For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.

38) $y = x^2 - 4x - 2$ at $(3, -5)$

39) $y = (3x + 6)^{\frac{1}{2}}$ at $(1, 3)$

40) $y = (x + 1)^{\frac{1}{3}}$ at $(-2, -1)$

41) $y = \frac{3}{x - 1}$ at $\left(-1, -\frac{3}{2}\right)$

For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

42)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	3	−1
2	3	1	2	−1
3	4	0	1	0
4	3	−1	2	1

Part 1) Given $h_1(x) = f(x) + g(x)$, find $h_1'(4)$
Part 2) Given $h_2(x) = f(x) - g(x)$, find $h_2'(4)$
Part 3) Given $h_3(x) = f(x) \cdot g(x)$, find $h_3'(3)$
Part 4) Given $h_4(x) = \frac{f(x)}{g(x)}$, find $h_4'(4)$
Part 5) Given $h_5(x) = (f(x))^2$, find $h_5'(3)$
Part 6) Given $h_6(x) = f(g(x))$, find $h_6'(1)$

43)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	2	2	2
2	3	$\frac{3}{2}$	4	$\frac{1}{2}$
3	4	0	3	−1
4	3	−1	2	−1

Part 1) Given $h_1(x) = f(x) + g(x)$, find $h_1'(4)$
Part 2) Given $h_2(x) = f(x) - g(x)$, find $h_2'(2)$
Part 3) Given $h_3(x) = f(x) \cdot g(x)$, find $h_3'(4)$
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Part 6) Given $h_6(x) = f(g(x))$, find $h_6'(4)$

44) List reasons a function might not be differentiable at a point. Give examples of each function and graph you use.

45) What is the Limit Existence Theorem?

46) What is local linearity?

47) In order for a function to be differentiable at a point, it must (a)_________________ and (b)_________________

Evaluate each limit.

48) $\lim_{x \to -1^-} f(x)$, $f(x) = \begin{cases} -2x + 2, & x < -1 \\ -x^2 + 2x - 2, & x \ge -1 \end{cases}$

49) $\lim_{x \to -2} f(x)$, $f(x) = \begin{cases} -2x - 6, & x \le -2 \\ -x^2 - 4x - 3, & x > -2 \end{cases}$

50) $\lim_{x \to 1^-} f(x),\ f(x) = \begin{cases} \dfrac{x}{2} - \dfrac{1}{2}, & x \le 1 \\ -x + 4, & x > 1 \end{cases}$

51) $\lim_{x \to -1} f(x),\ f(x) = \begin{cases} x^2 + 4x + 3, & x \le -1 \\ -x - 3, & x > -1 \end{cases}$

52) $\lim_{x \to 3} f(x),\ f(x) = \begin{cases} -\dfrac{x}{2} + \dfrac{3}{2}, & x < 3 \\ 2x - 6, & x \ge 3 \end{cases}$

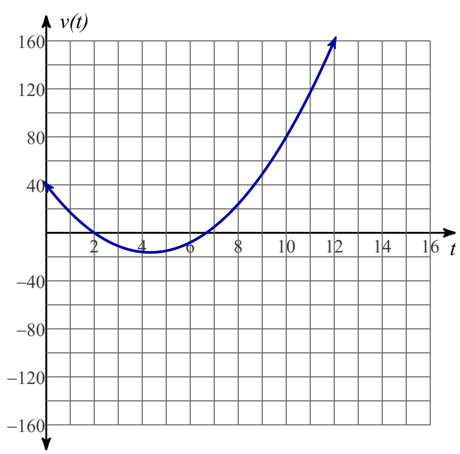
53) $\lim_{x \to 2} f(x),\ f(x) = \begin{cases} -x - 3, & x \le 2 \\ -x^2 - 1, & x > 2 \end{cases}$

54) $\lim_{x \to 3} -\dfrac{x^2 - 5x + 6}{x - 3}$

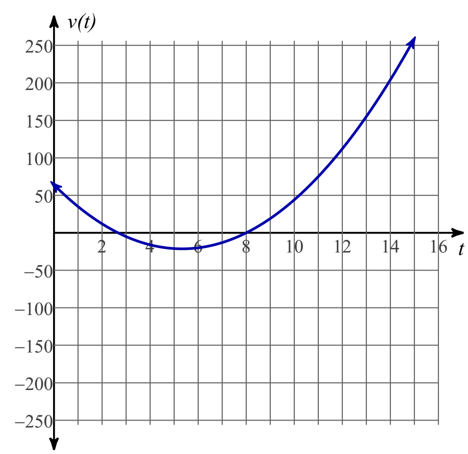
55) $\lim_{x \to -2} f(x),\ f(x) = \begin{cases} 2x + 1, & x \ne -2 \\ -5, & x = -2 \end{cases}$

A particle moves along a horizontal line. Its velocity function is $v(t)$ for $t \ge 0$. For each problem, find the times t when the particle changes directions. The graph of $v(t)$ is provided.

56) $v(t) = 3t^2 - 26t + 40$



57) $v(t) = 3t^2 - 32t + 64$



58) Find the equation of the tangent to the curve of f(x) = $\sqrt{-3x + 6}$ that has slope $-\dfrac{1}{2}$.

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

59) $y = -2x^2 - 8x - 9$ at $(-1, -3)$

60) $y = -(2x + 2)^{\frac{1}{2}}$ at $(1, -2)$

61) $y = -\dfrac{x^2}{2} + 3x - \dfrac{11}{2}$ at $\left(0, -\dfrac{11}{2}\right)$

62) $y = -\dfrac{x^2}{4x + 4}$ at $\left(2, -\dfrac{1}{3}\right)$

Answers to

1) 0
2) $\frac{25}{26}$
3) 0
4) 1
5) ∞
6) $-\infty$
7) -1
8) 1
9) -1
10) 5
11) -3
12) 1

13) $\frac{dy}{dx} = \frac{4}{5x^{\frac{3}{5}}} + \frac{2}{5x^{\frac{4}{5}}} + \frac{25}{3x^6}$

14) $\frac{dy}{dx} = -4x + \frac{4}{x^4} - \frac{5}{x^5}$

15) $\frac{dy}{dx} = 20x^3 - 5x^2 - \frac{4x^{\frac{1}{3}}}{3}$

16) $\frac{dy}{dx} = (3x^3 + 2) \cdot -10x^4 - 2x^5 \cdot 9x^2$
$= -48x^7 - 20x^4$

17) $\frac{dy}{dx} = (2x^5 + 3) \cdot 12x^3 + (3x^4 - 3) \cdot 10x^4$
$= 54x^8 - 30x^4 + 36x^3$

18) $\frac{dy}{dx} = \left(-5x^{\frac{1}{3}} - 2\right) \cdot -12x^2 + (-4x^3 - 5) \cdot -\frac{5}{3}x^{-\frac{2}{3}}$
$= \frac{200x^{\frac{7}{3}}}{3} + 24x^2 + \frac{25}{3x^{\frac{2}{3}}}$

19) $\frac{dy}{dx} = \frac{(3x^4 - 5)(8x^3 + 4x) - (2x^4 + 2x^2) \cdot 12x^3}{(3x^4 - 5)^2}$
$= \frac{-12x^5 - 40x^3 - 20x}{9x^8 - 30x^4 + 25}$

20) $\frac{dy}{dx} = \frac{(4x^4 - 5)(8x^3 + 15x^2) - (2x^4 + 5x^3) \cdot 16x^3}{(4x^4 - 5)^2}$
$= \frac{-20x^6 - 40x^3 - 75x^2}{16x^8 - 40x^4 + 25}$

21) $\frac{dy}{dx} = 5(2x^3 - 1)^4 \cdot 6x^2$
$= 30x^2(2x^3 - 1)^4$

22) $\frac{dy}{dx} = \frac{1}{3}(-2x^3 + 5)^{-\frac{2}{3}} \cdot -6x^2$
$= -\frac{2x^2}{(-2x^3 + 5)^{\frac{2}{3}}}$

23) $\frac{dy}{dx} = (3x^5 + 1) \cdot \frac{1}{2}(-x^2 + 2)^{-\frac{1}{2}} \cdot -2x + (-x^2 + 2)^{\frac{1}{2}} \cdot 15x^4$
$= \frac{x(-18x^5 + 30x^3 - 1)}{(-x^2 + 2)^{\frac{1}{2}}}$

24) Average: -3
25) Average: 0
26) -7
27) 1
28) $y = \frac{3}{2}x - 4$
29) $y = -\frac{1}{7}x - \frac{8}{7}$
30) $(2, -1)$
31) $\left(0, -\frac{1}{2}\right)$
32) $(0, 2), (2, 6)$
33) $(0, 0), (3, -3)$
34) $v(8) = 0, a(8) = 24$
35) $v(2) = -64, a(2) = 4$
36) Changes direction at: $t = \{8\}$
37) Changes direction at: $t = \left\{\frac{4}{3}, 6\right\}$
38) $y = -\frac{1}{2}x - \frac{7}{2}$
39) $y = -2x + 5$
40) $y = -3x - 7$
41) $y = \frac{4}{3}x - \frac{1}{6}$

42) $h_1'(4) = 0$
$h_2'(4) = -2$
$h_3'(3) = 0$
$h_4'(4) = -\frac{5}{4}$
$h_5'(3) = 0$
$h_6'(1) = 0$

43) $h_1'(4) = -2$
$h_2'(2) = 1$
$h_3'(4) = -5$
$h_4'(4) = \frac{1}{4}$
$h_5'(3) = 0$
$h_6'(4) = -\frac{3}{2}$

44)

45)

46)

47)

48) 4

49) Does not exist.

50) 0

51) Does not exist.

52) 0

53) −5

54) −1

55) −3

56) Changes direction at: $t = \left\{2, \frac{20}{3}\right\}$

57) Changes direction at: $t = \left\{\frac{8}{3}, 8\right\}$

58)

59) $y = -4x - 7$

60) $y = -\frac{1}{2}x - \frac{3}{2}$

61) $y = 3x - \frac{11}{2}$

62) $y = -\frac{2}{9}x + \frac{1}{9}$

