Calculus - NO CALCULATOR!!! Name_____

Avg. Value, MVTI, 2nd FTC Review

For each problem, find the average value of the function over the given interval.

1) $f(x) = -\cos x;\ [-\frac{\pi}{4}, \frac{\pi}{3}]$

$$f_{avg} = \frac{1}{\frac{\pi}{3}+\frac{\pi}{4}} \int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} -\cos x\,dx$$

$$= \frac{1}{\frac{7\pi}{12}} \left(-\sin x\right)\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \frac{-12}{7\pi}\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\right) = \frac{-12}{7\pi}\left(\frac{\sqrt{3}+\sqrt{2}}{2}\right)$$

$$= \boxed{\frac{-6\sqrt{3}-6\sqrt{2}}{7\pi}}$$

2) $f(x) = -\frac{2}{x};\ [-5, -2]$

$$f_{avg} = \frac{1}{3}\int_{-5}^{-2} -\frac{2}{x}\,dx = -\frac{2}{3}\int_{-5}^{-2} \frac{1}{x}\,dx$$

$$= -\frac{2}{3}\ln|x|\Big|_{-5}^{-2} = -\frac{2}{3}(\ln 2 - \ln 5)$$

$$= \boxed{\frac{-2\ln 2 + 2\ln 5}{3}}$$

For each problem, find the average value of the function over the given interval. Then, find the values of *c* that satisfy the Mean Value Theorem for Integrals.

3) $f(x) = 2x + 1;\ [-2, 3]$

$$f_{avg} = \frac{1}{5}\int_{-2}^{3} (2x+1)\,dx = \frac{1}{5}\left(x^2 + x\right)\Big|_{-2}^{3}$$

$$= \frac{1}{5}\left(9+3-(4-2)\right) = \boxed{2}$$

MVTI:

$$2 = 2x + 1$$

$$x = \frac{1}{2}$$

$$c = \frac{1}{2}$$

4) $f(x) = 2x^2 - 16x + 27;\ [2, 5]$

$$f_{avg} = \frac{1}{3}\int_{2}^{5} (2x^2 - 16x + 27)\,dx$$

$$= \frac{1}{3}\left(\frac{2}{3}x^3 - 8x^2 + 27x\right)\Big|_{2}^{5} = \boxed{-3}$$

MVTI:

$$-3 = 2x^2 - 16x + 27$$

$$0 = 2x^2 - 16x + 30$$

$$0 = 2(x^2 - 8x + 15)$$

$$0 = 2(x-5)(x-3)$$

$$\boxed{x = 3, 5}$$

5) $f(x) = \frac{3}{(x+2)^2};\ [-1, 2]$

$$f_{avg} = \frac{1}{3}\int_{-1}^{2} \frac{3}{(x+2)^2}\,dx = \int_{-1}^{2} (x+2)^{-2}\,dx$$

$$= -(x+2)^{-1}\Big|_{-1}^{2} = \frac{-1}{x+2}\Big|_{-1}^{2} = -\frac{1}{4} + 1 = \boxed{\frac{3}{4}}$$

MVTI:

$$\frac{3}{4} = \frac{3}{(x+2)^2}$$

$$3(x+2)^2 = 12$$

$$(x+2)^2 = 4$$

$$x+2 = \pm 2$$

$$x = -2 \pm 2$$

$$x = 0, -4 \rightarrow$$ (-4 is not in interval)

$$\boxed{x = 0}$$

6) $f(x) = 2(x+1)^{\frac{1}{2}};\ [-1, 2]$

$$f_{avg} = \frac{1}{3}\int_{-1}^{2} 2(x+1)^{\frac{1}{2}}\,dx = \frac{2}{3}\left(\frac{2}{3}(x+1)^{\frac{3}{2}}\right)\Big|_{-1}^{2}$$

$$= \frac{4}{9}(x+1)^{\frac{3}{2}}\Big|_{-1}^{2} = \frac{4}{9}(3\sqrt{3}) = \boxed{\frac{4\sqrt{3}}{3}}$$

MVTI:

$$\frac{4\sqrt{3}}{3} = 2(x+1)^{1/2}$$

$$\frac{2\sqrt{3}}{3} = (x+1)^{\frac{1}{2}}$$

$$\frac{12}{9} = x+1$$

$$\boxed{x = \frac{1}{3}}$$

For each problem, find $F'(x)$. $\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)\,dt = f(b(x))\,b'(x) - f(a(x))\,a'(x)$

7) $F(x) = \int_0^{3x} (t^2 - 6t + 6)\,dt$

$F'(x) = \left((3x)^2 - 6(3x) + 6\right)(3) - 0$

$= \boxed{27x^2 - 54x + 18}$

8) $F(x) = \int_{-\frac{\pi}{4}}^{x^2} -\sec t \tan t\,dt$

$F'(x) = (-\sec(x^2)\tan(x^2))(2x) - 0$

$= \boxed{-2x\sec(x^2)\tan(x^2)}$

9) $F(x) = \int_{-3}^{2x} 3e^t\,dt$

$F'(x) = (3e^{2x})(2) - 0$

$= \boxed{6e^{2x}}$

10) $F(x) = \int_x^{x^2} (-t^2 + 4t + 2)\,dt$

$F'(x) = (-(x^2)^2 + 4(x^2) + 2)(2x) - (-x^2 + 4x + 2)(1)$

$= -2x^5 + 8x^3 + 4x + x^2 - 4x - 2$

$= \boxed{-2x^5 + 8x^3 + x^2 - 2}$

11) $F(x) = \int_x^{x^2} -2\csc t\cot t\,dt$

$F'(x) = (-2\csc(x^2)\cot(x^2))(2x) - (-2\csc x\cot x)(1)$

$= \boxed{-4x\csc(x^2)\cot(x^2) + 2\csc x\cot x}$

12) $F(x) = \int_x^{2x} \frac{2}{t}\,dt$

$F'(x) = \left(\frac{2}{2x}\right)(2) - \left(\frac{2}{x}\right)(1)$

$= \boxed{0}$