Calculus Name_____

Additional Test Review

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the maximum speed and times t when this speed occurs, the displacement of the particle, and the distance traveled by the particle over the given interval.

1) $s(t) = -t^3 + 18t^2 - 81t;\ 3 \leq t \leq 7$

2) $s(t) = t^4 - 8t^3;\ 0 \leq t \leq 8$

3) $s(t) = t^3 - 24t^2 + 144t;\ 4 \leq t \leq 6$

4) $s(t) = t^4 - 8t^3;\ 3 \leq t \leq 8$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the times t when the particle changes directions, the intervals of time when the particle is moving left and moving right, and the intervals of time when the particle is slowing down and speeding up.

5) $s(t) = -t^3 + 16t^2 - 64t$

6) $s(t) = t^4 - 15t^3$

For each problem, find the: x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

7) $y = x^3 - 2x^2$

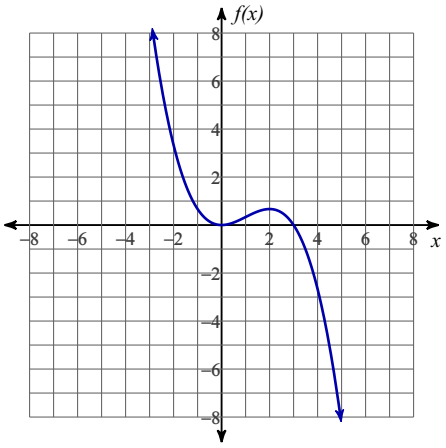
8) $y = -x^4 + 2x^2 - 1$

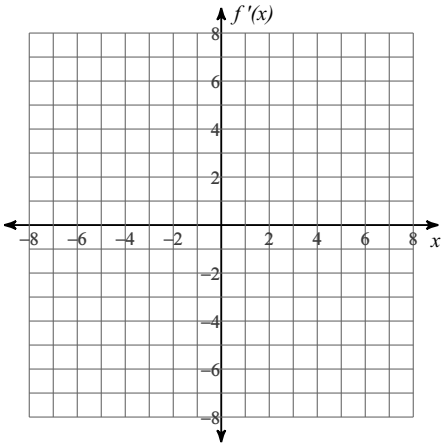
9) $y = \dfrac{1}{x - 1}$

10) $y = (x + 5)^{\frac{2}{3}}$

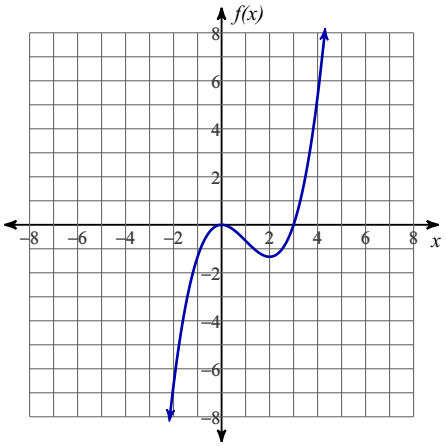
Given the graph of $f(x)$, sketch an approximate graph of $f'(x)$.

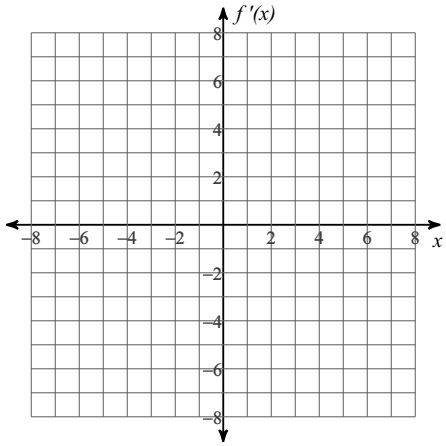
11)





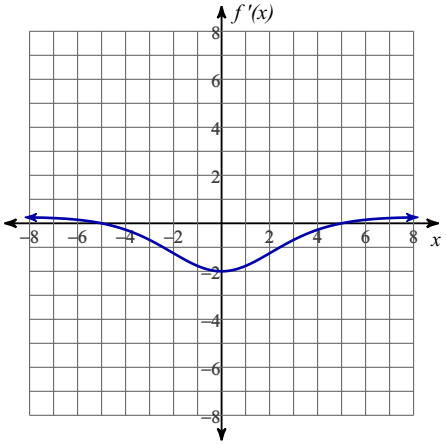
12)

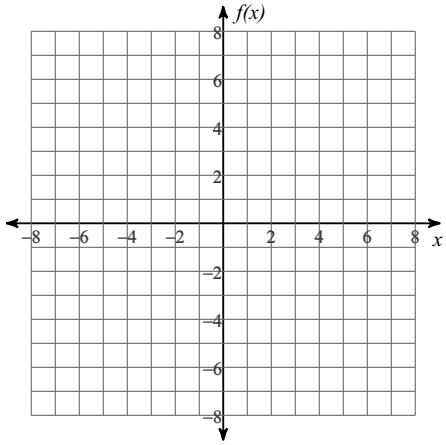




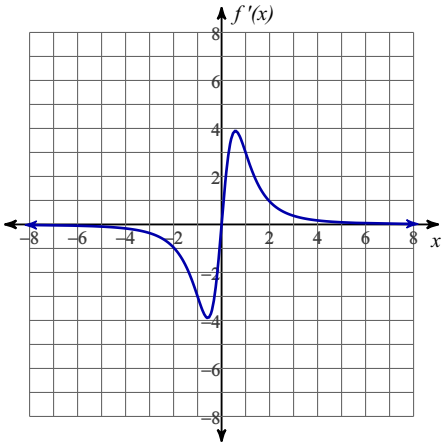
Given the graph of $f'(x)$, sketch a possible graph of $f(x)$.

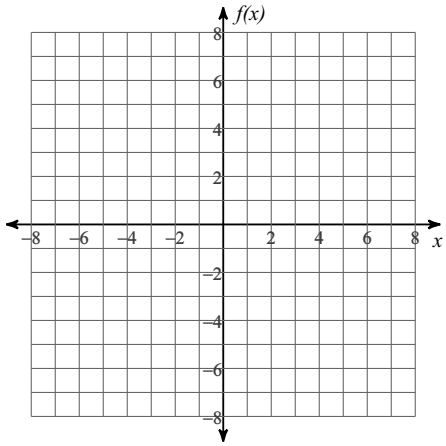
13)





14)





Answers to Additional Test Review

1) Maximum speed: 27 at $t = \{6\}$
Displacement: 80
Distance traveled: 80

2) Maximum speed: 512 at $t = \{8\}$
Displacement: 0
Distance traveled: 864

3) Maximum speed: 36 at $t = \{6\}$
Displacement: −40
Distance traveled: 40

4) Maximum speed: 512 at $t = \{8\}$
Displacement: 135
Distance traveled: 729

5) Changes direction at: $t = \{\frac{8}{3}, 8\}$,

Moving left: $0 \le t < \frac{8}{3}$, $t > 8$,

Moving right: $\frac{8}{3} < t < 8$

Slowing down: $0 \le t < \frac{8}{3}$, $\frac{16}{3} < t < 8$,

Speeding up: $\frac{8}{3} < t < \frac{16}{3}$, $t > 8$

6) Changes direction at: $t = \{\frac{45}{4}\}$,

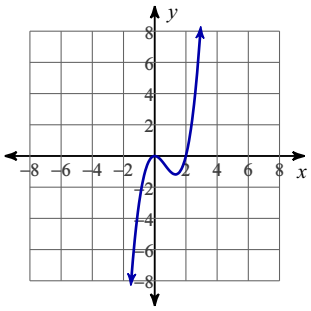
Moving left: $0 < t < \frac{45}{4}$,

Moving right: $t > \frac{45}{4}$

Slowing down: $\frac{15}{2} < t < \frac{45}{4}$,

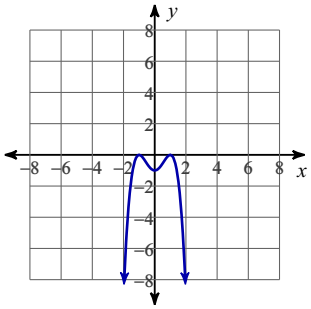
Speeding up: $0 < t < \frac{15}{2}$, $t > \frac{45}{4}$

7)



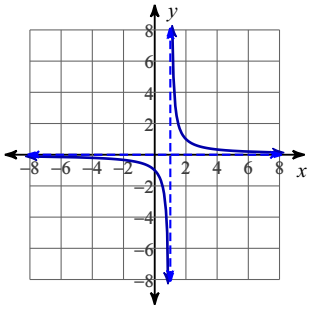
Critical points at: $x = 0, \frac{4}{3}$
Increasing: $(-\infty, 0), \left(\frac{4}{3}, \infty\right)$ Decreasing: $\left(0, \frac{4}{3}\right)$
Inflection point at: $x = \frac{2}{3}$
Concave up: $\left(\frac{2}{3}, \infty\right)$ Concave down: $\left(-\infty, \frac{2}{3}\right)$
Relative minimum: $\left(\frac{4}{3}, -\frac{32}{27}\right)$ Relative maximum: $(0, 0)$

8)



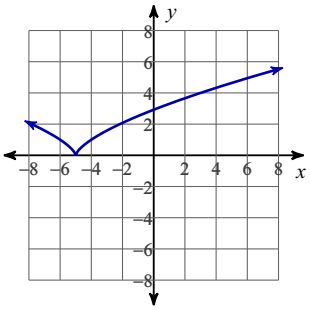
Critical points at: $x = -1, 0, 1$
Increasing: $(-\infty, -1), (0, 1)$ Decreasing: $(-1, 0), (1, \infty)$
Inflection points at: $x = -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}$
Concave up: $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{3}}{3}\right), \left(\frac{\sqrt{3}}{3}, \infty\right)$
Relative minimum: $(0, -1)$ Relative maxima: $(-1, 0), (1, 0)$

9)



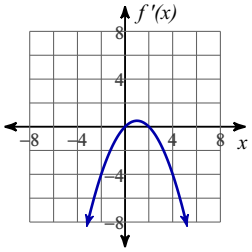
No critical points exist.
Increasing: No intervals exist. Decreasing: $(-\infty, 1), (1, \infty)$
No inflection points exist.
Concave up: $(1, \infty)$ Concave down: $(-\infty, 1)$
No relative minima. No relative maxima.

10)

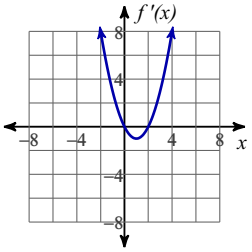


Critical point at: $x = -5$
Increasing: $(-5, \infty)$ Decreasing: $(-\infty, -5)$
No inflection points exist.
Concave up: No intervals exist. Concave down: $(-\infty, -5), (-5, \infty)$
Relative minimum: $(-5, 0)$ No relative maxima.

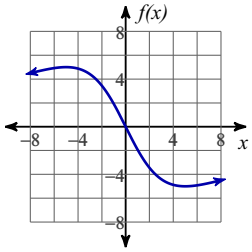
11)



12)



13)



14)