Calculus Name________________________________

Applications of Trigonometric Derivatives #1

For each problem, find the derivative of the function at the given value.

1) $y = -\sec(x)$ at $x = \pi$

2) $y = -2\cos(2x)$ at $x = 0$

3) $y = -\cot(2x)$ at $x = -\frac{5\pi}{6}$

4) $y = -\csc(2x)$ at $x = \frac{\pi}{3}$

5) $y = -2\tan(2x)$ at $x = \pi$

6) $y = 2\sin(x)$ at $x = -\frac{\pi}{4}$

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

7) $y = \sin(2x)$ at $\left(\frac{\pi}{2}, 0\right)$

8) $y = -\cos(2x)$ at $(-\pi, -1)$

9) $y = -\csc(x)$ at $\left(\frac{\pi}{3}, -\frac{2\sqrt{3}}{3}\right)$

10) $y = -2\tan(2x)$ at $(-\pi, 0)$

11) $y = \cot(2x)$ at $\left(\frac{3\pi}{4}, 0\right)$

12) $y = \sec(x)$ at $\left(-\frac{\pi}{6}, \frac{2\sqrt{3}}{3}\right)$

For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.

13) $y = -2\sin(x)$ at $(0, 0)$

14) $y = -2\cos(x)$ at $\left(-\frac{5\pi}{6}, \sqrt{3}\right)$

15) $y = \csc(x)$ at $\left(-\frac{\pi}{2}, -1\right)$

16) $y = -2\sec(x)$ at $\left(-\frac{5\pi}{6}, \frac{4\sqrt{3}}{3}\right)$

17) $y = -\tan(x)$ at $(0, 0)$

18) $y = -2\cot(2x)$ at $\left(\frac{2\pi}{3}, -\frac{2\sqrt{3}}{3}\right)$

For each problem, find the points where the tangent line to the function is horizontal. Indicate if no horizontal tangent line exists.

19) $y = 2\cos(x)$; $[-\pi, \pi]$

20) $y = \sin(x)$; $[-\pi, \pi]$

21) $y = -2\sec(x)$; $[-\pi, \pi]$

22) $y = \sin(2x)$; $[-\pi, \pi]$

23) $y = 2\csc(2x)$; $[-\pi, \pi]$

24) $y = 2\cot(x)$; $[-\pi, \pi]$

25) $y = 2\tan(x);\ [-\pi, \pi]$

26) $y = -\cos(2x);\ [-\pi, \pi]$

For each problem, find all points of absolute minima and maxima on the given interval.

27) $y = -\sin(2x);\ [0, \frac{\pi}{4}]$

28) $y = -\csc(2x);\ [\frac{\pi}{4}, \frac{\pi}{3}]$

29) $y = -2\cot(x);\ [-\frac{\pi}{6}, \frac{\pi}{4}]$

30) $y = \tan(2x);\ [-\frac{\pi}{6}, \frac{\pi}{6}]$

31) $y = 2\cos(2x);\ [0, \frac{3\pi}{4}]$

32) $y = 2\sec(2x);\ [-\frac{\pi}{2}, -\frac{\pi}{3}]$

For each problem, find the x-coordinates of all points of inflection, find all discontinuities, and find the open intervals where the function is concave up and concave down.

33) $y = -\cot(2x);\ [-\pi, \pi]$

34) $y = 2\cos(2x);\ [-\pi, \pi]$

35) $y = \sec(2x)$; $[-\pi, \pi]$

36) $y = \tan(x)$; $[-\pi, \pi]$

37) $y = -\csc(2x)$; $[-\pi, \pi]$

38) $y = -2\sin(x)$; $[-\pi, \pi]$

For each problem, find the values of *c* that satisfy Rolle's Theorem.

39) $y = -2\sin(2x)$; $[-\pi, \pi]$

40) $y = -\cos(2x)$; $[-\pi, \pi]$

41) $y = -\sin(x)$; $[-\pi, \pi]$

42) $y = 2\cos(2x)$; $[-\pi, \pi]$

Evaluate each limit. Use L'Hôpital's Rule if it can be applied. If it cannot be applied, evaluate using another method and write a * next to your answer.

43) $\lim_{x \to 0} \frac{\sin(3x)}{\sin(5x)}$

44) $\lim_{x \to 0} \frac{4x^2}{1 - \cos(3x)}$

45) $\lim_{x \to 0} \frac{2(e^x - x)}{1 - \cos x}$

46) $\lim_{x \to 0} \frac{1 - \cos(3x)}{\cos(2x) - 1}$

47) $\lim_{x \to 0} \frac{\sin(5x)}{2x}$

48) $\lim_{x \to 0} \frac{2(e^x - e^{-x})}{\sin(2x)}$

For each problem, find all points of relative minima and maxima.

49) $y = -\csc(2x);\ [-\pi, \pi]$

50) $y = -2\cos(x);\ [-\pi, \pi]$

51) $y = \sin(x);\ [-\pi, \pi]$

52) $y = -\tan(2x);\ [-\pi, \pi]$

53) $y = -2\sec(2x);\ [-\pi, \pi]$

54) $y = -\cot(2x);\ [-\pi, \pi]$

55) .

Name________________________

Applications of Trigonometric Derivatives #1

For each problem, find the derivative of the function at the given value.

1) $y = -\sec(x)$ at $x = \pi$

$\left.\frac{dy}{dx}\right|_{x=\pi} = 0$

2) $y = -2\cos(2x)$ at $x = 0$

$\left.\frac{dy}{dx}\right|_{x=0} = 0$

3) $y = -\cot(2x)$ at $x = -\frac{5\pi}{6}$

$\left.\frac{dy}{dx}\right|_{x=-\frac{5\pi}{6}} = \frac{8}{3}$

4) $y = -\csc(2x)$ at $x = \frac{\pi}{3}$

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{3}} = -\frac{4}{3}$

5) $y = -2\tan(2x)$ at $x = \pi$

$\left.\frac{dy}{dx}\right|_{x=\pi} = -4$

6) $y = 2\sin(x)$ at $x = -\frac{\pi}{4}$

$\left.\frac{dy}{dx}\right|_{x=-\frac{\pi}{4}} = \sqrt{2}$

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

7) $y = \sin(2x)$ at $\left(\frac{\pi}{2}, 0\right)$

$y = -2x + \pi$

8) $y = -\cos(2x)$ at $(-\pi, -1)$

$y = -1$

9) $y = -\csc(x)$ at $\left(\frac{\pi}{3}, -\frac{2\sqrt{3}}{3}\right)$

$y = \frac{2}{3}x + \frac{-6\sqrt{3} - 2\pi}{9}$

10) $y = -2\tan(2x)$ at $(-\pi, 0)$

$y = -4x - 4\pi$

11) $y = \cot(2x)$ at $\left(\frac{3\pi}{4}, 0\right)$

$y = -2x + \frac{3\pi}{2}$

12) $y = \sec(x)$ at $\left(-\frac{\pi}{6}, \frac{2\sqrt{3}}{3}\right)$

$y = -\frac{2}{3}x + \frac{6\sqrt{3} - \pi}{9}$

For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.

13) $y = -2\sin(x)$ at $(0, 0)$

$y = \frac{1}{2}x$

14) $y = -2\cos(x)$ at $\left(-\frac{5\pi}{6}, \sqrt{3}\right)$

$y = x + \sqrt{3} + \frac{5\pi}{6}$

15) $y = \csc(x)$ at $\left(-\frac{\pi}{2}, -1\right)$

Normal line is vertical line at $x = -\frac{\pi}{2}$

16) $y = -2\sec(x)$ at $\left(-\frac{5\pi}{6}, \frac{4\sqrt{3}}{3}\right)$

$y = -\frac{3}{4}x + \frac{32\sqrt{3} - 15\pi}{24}$

17) $y = -\tan(x)$ at $(0, 0)$

$y = x$

18) $y = -2\cot(2x)$ at $\left(\frac{2\pi}{3}, -\frac{2\sqrt{3}}{3}\right)$

$y = -\frac{3}{16}x + \frac{-16\sqrt{3} + 3\pi}{24}$

For each problem, find the points where the tangent line to the function is horizontal. Indicate if no horizontal tangent line exists.

19) $y = 2\cos(x)$; $[-\pi, \pi]$

$(-\pi, -2), (0, 2), (\pi, -2)$

20) $y = \sin(x)$; $[-\pi, \pi]$

$\left(-\frac{\pi}{2}, -1\right), \left(\frac{\pi}{2}, 1\right)$

21) $y = -2\sec(x)$; $[-\pi, \pi]$

$(-\pi, 2), (0, -2), (\pi, 2)$

22) $y = \sin(2x)$; $[-\pi, \pi]$

$\left(-\frac{3\pi}{4}, 1\right), \left(-\frac{\pi}{4}, -1\right), \left(\frac{\pi}{4}, 1\right), \left(\frac{3\pi}{4}, -1\right)$

23) $y = 2\csc(2x)$; $[-\pi, \pi]$

$\left(-\frac{3\pi}{4}, 2\right), \left(-\frac{\pi}{4}, -2\right), \left(\frac{\pi}{4}, 2\right), \left(\frac{3\pi}{4}, -2\right)$

24) $y = 2\cot(x)$; $[-\pi, \pi]$

No horizontal tangent line exits.

25) $y = 2\tan(x)$; $[-\pi, \pi]$

No horizontal tangent line exits.

26) $y = -\cos(2x)$; $[-\pi, \pi]$

$(-\pi, -1), \left(-\frac{\pi}{2}, 1\right), (0, -1), \left(\frac{\pi}{2}, 1\right), (\pi, -1)$

For each problem, find all points of absolute minima and maxima on the given interval.

27) $y = -\sin(2x)$; $[0, \frac{\pi}{4}]$

Absolute minimum: $\left(\frac{\pi}{4}, -1\right)$

Absolute maximum: $(0, 0)$

28) $y = -\csc(2x)$; $[\frac{\pi}{4}, \frac{\pi}{3}]$

Absolute minimum: $\left(\frac{\pi}{3}, -\frac{2\sqrt{3}}{3}\right)$

Absolute maximum: $\left(\frac{\pi}{4}, -1\right)$

29) $y = -2\cot(x)$; $[-\frac{\pi}{6}, \frac{\pi}{4}]$

No absolute minima.
No absolute maxima.

30) $y = \tan(2x)$; $[-\frac{\pi}{6}, \frac{\pi}{6}]$

Absolute minimum: $\left(-\frac{\pi}{6}, -\sqrt{3}\right)$

Absolute maximum: $\left(\frac{\pi}{6}, \sqrt{3}\right)$

31) $y = 2\cos(2x)$; $[0, \frac{3\pi}{4}]$

Absolute minimum: $\left(\frac{\pi}{2}, -2\right)$

Absolute maximum: $(0, 2)$

32) $y = 2\sec(2x)$; $[-\frac{\pi}{2}, -\frac{\pi}{3}]$

Absolute minimum: $\left(-\frac{\pi}{3}, -4\right)$

Absolute maximum: $\left(-\frac{\pi}{2}, -2\right)$

For each problem, find the x-coordinates of all points of inflection, find all discontinuities, and find the open intervals where the function is concave up and concave down.

33) $y = -\cot(2x)$; $[-\pi, \pi]$

Inflection points at: $x = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$ Discontinuities at: $x = -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi$

Concave up: $\left(-\frac{3\pi}{4}, -\frac{\pi}{2}\right), \left(-\frac{\pi}{4}, 0\right), \left(\frac{\pi}{4}, \frac{\pi}{2}\right), \left(\frac{3\pi}{4}, \pi\right)$ Concave down: $\left(-\pi, -\frac{3\pi}{4}\right), \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right), \left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

34) $y = 2\cos(2x)$; $[-\pi, \pi]$

Inflection points at: $x = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$ No discontinuities exist.

Concave up: $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ Concave down: $\left(-\pi, -\frac{3\pi}{4}\right), \left(-\frac{\pi}{4}, \frac{\pi}{4}\right), \left(\frac{3\pi}{4}, \pi\right)$

35) $y = \sec(2x);\ [-\pi, \pi]$

No inflection points exist. Discontinuities at: $x = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$

Concave up: $\left(-\pi, -\frac{3\pi}{4}\right), \left(-\frac{\pi}{4}, \frac{\pi}{4}\right), \left(\frac{3\pi}{4}, \pi\right)$ Concave down: $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

36) $y = \tan(x);\ [-\pi, \pi]$

Inflection points at: $x = -\pi, 0, \pi$ Discontinuities at: $x = -\frac{\pi}{2}, \frac{\pi}{2}$

Concave up: $\left(-\pi, -\frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$ Concave down: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, \pi\right)$

37) $y = -\csc(2x);\ [-\pi, \pi]$

No inflection points exist. Discontinuities at: $x = -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi$

Concave up: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, \pi\right)$ Concave down: $\left(-\pi, -\frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$

38) $y = -2\sin(x);\ [-\pi, \pi]$

Inflection points at: $x = -\pi, 0, \pi$ No discontinuities exist.
Concave up: $(0, \pi)$ Concave down: $(-\pi, 0)$

For each problem, find the values of *c* that satisfy Rolle's Theorem.

39) $y = -2\sin(2x);\ [-\pi, \pi]$

$\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}\right\}$

40) $y = -\cos(2x);\ [-\pi, \pi]$

$\left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

41) $y = -\sin(x);\ [-\pi, \pi]$

$\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$

42) $y = 2\cos(2x);\ [-\pi, \pi]$

$\left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

Evaluate each limit. Use L'Hôpital's Rule if it can be applied. If it cannot be applied, evaluate using another method and write a * next to your answer.

43) $\lim_{x \to 0} \frac{\sin(3x)}{\sin(5x)}$

$\frac{3}{5}$

44) $\lim_{x \to 0} \frac{4x^2}{1 - \cos(3x)}$

$\frac{8}{9}$

45) $\lim_{x \to 0} \frac{2(e^x - x)}{1 - \cos x}$

∞ *

46) $\lim_{x \to 0} \frac{1 - \cos(3x)}{\cos(2x) - 1}$

$-\frac{9}{4}$

47) $\lim_{x \to 0} \frac{\sin(5x)}{2x}$

$\frac{5}{2}$

48) $\lim_{x \to 0} \frac{2(e^x - e^{-x})}{\sin(2x)}$

2

For each problem, find all points of relative minima and maxima.

49) $y = -\csc(2x);\ [-\pi, \pi]$

Relative minima: $\left(-\frac{\pi}{4}, 1\right), \left(\frac{3\pi}{4}, 1\right)$

Relative maxima: $\left(-\frac{3\pi}{4}, -1\right), \left(\frac{\pi}{4}, -1\right)$

50) $y = -2\cos(x);\ [-\pi, \pi]$

Relative minimum: $(0, -2)$
Relative maxima: $(-\pi, 2), (\pi, 2)$

51) $y = \sin(x);\ [-\pi, \pi]$

Relative minimum: $\left(-\frac{\pi}{2}, -1\right)$

Relative maximum: $\left(\frac{\pi}{2}, 1\right)$

52) $y = -\tan(2x);\ [-\pi, \pi]$

No relative minima.
No relative maxima.

53) $y = -2\sec(2x);\ [-\pi, \pi]$

Relative minima: $\left(-\frac{\pi}{2}, 2\right), \left(\frac{\pi}{2}, 2\right)$

Relative maxima: $(-\pi, -2), (0, -2), (\pi, -2)$

54) $y = -\cot(2x);\ [-\pi, \pi]$

No relative minima.
No relative maxima.

55) .

