

Horizontal Tangents & Review

For each problem, find the points where the tangent line to the function is horizontal. Indicate if no horizontal tangent line exists.

$$1) \ f(x) = \frac{3}{x-1}$$

No horizontal tangent line exists.

$$2) \ f(x) = -\frac{1}{x^2 - 1}$$

(0, 1)

$$3) \ f(x) = x^3 - 3x^2 + 6$$

(0, 6), (2, 2)

$$4) \ f(x) = -\frac{2}{x-2}$$

No horizontal tangent line exists.

$$5) \ f(x) = \frac{1}{x+2}$$

No horizontal tangent line exists.

$$6) \ f(x) = -x^3 - 3x^2 - 27x - 3$$

No horizontal tangent line exists.

$$7) \ f(x) = \frac{3}{x^2 - 4}$$

$\left(0, -\frac{3}{4}\right)$

$$8) \ f(x) = -\frac{9x}{x^2 + 9}$$

$\left(-3, \frac{3}{2}\right), \left(3, -\frac{3}{2}\right)$

$$9) \ f(x) = -x^3 + 3x^2 + 2$$

(0, 2), (2, 6)

$$10) \ f(x) = (3x + 3)^{\frac{1}{2}}$$

No horizontal tangent line exists.

$$11) \ f(x) = -\frac{x^2}{2x + 4}$$

(-4, 4), (0, 0)

$$12) \ f(x) = -(x + 2)^{\frac{1}{3}}$$

No horizontal tangent line exists.

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

13) $f(x) = x^3 - 2x^2 + 3$ at $(0, 3)$
 $y = 3$

14) $f(x) = \frac{2}{x-2}$ at $(-2, -\frac{1}{2})$
 $y = -\frac{1}{8}x - \frac{3}{4}$

Differentiate each function with respect to x .

15) $y = \frac{4x^4 - 2}{5x^4 + 5}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(5x^4 + 5) \cdot 16x^3 - (4x^4 - 2) \cdot 20x^3}{(5x^4 + 5)^2} \\ &= \frac{24x^3}{5x^8 + 10x^4 + 5}\end{aligned}$$

16) $y = \frac{5}{x^3 - 2}$

$$\begin{aligned}\frac{dy}{dx} &= -\frac{5 \cdot 3x^2}{(x^3 - 2)^2} \\ &= -\frac{15x^2}{x^6 - 4x^3 + 4}\end{aligned}$$

17) $y = 5x^2(5x^5 + 3)$

$$\begin{aligned}\frac{dy}{dx} &= 5x^2 \cdot 25x^4 + (5x^5 + 3) \cdot 10x \\ &= 175x^6 + 30x\end{aligned}$$

18) $y = 4x^3(-2x^3 + 4)$

$$\begin{aligned}\frac{dy}{dx} &= 4x^3 \cdot -6x^2 + (-2x^3 + 4) \cdot 12x^2 \\ &= -48x^5 + 48x^2\end{aligned}$$

19) $y = 2x^4 - 4x^2$

$$\frac{dy}{dx} = 8x^3 - 8x$$

20) $y = 5x^5 + 4x^4 - 4x$

$$\frac{dy}{dx} = 25x^4 + 16x^3 - 4$$

Use the definition of the derivative to find the derivative of each function with respect to x .

21) $y = 5x + 1$

$$\frac{dy}{dx} = 5$$

22) $y = 2x^2 + 4$

$$\frac{dy}{dx} = 4x$$

23) $y = -\frac{2}{2x - 5}$

$$\frac{dy}{dx} = \frac{4}{4x^2 - 20x + 25}$$

24) $y = \sqrt{-4x + 4}$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{-x + 1}}$$