

## Limits and Derivatives of Trig Functions

**Evaluate each limit.**

1)  $\lim_{x \rightarrow -\frac{3\pi}{4}} -\sec(x)$

2)  $\lim_{x \rightarrow -\pi} \cos(x)$

3)  $\lim_{x \rightarrow \frac{5\pi}{6}} \csc(2x)$

4)  $\lim_{x \rightarrow \frac{3\pi}{4}} -2\sin(2x)$

5)  $\lim_{x \rightarrow -\frac{\pi}{2}} -2\sin(x)$

6)  $\lim_{x \rightarrow -\frac{\pi}{6}} 2\cos(x)$

7)  $\lim_{x \rightarrow \frac{\pi}{3}} -\tan(x)$

8)  $\lim_{x \rightarrow -\pi} \sec(2x)$

9)  $\lim_{x \rightarrow \frac{3\pi}{4}} \cot(x)$

10)  $\lim_{x \rightarrow \frac{5\pi}{6}} -\sin(2x)$

**Differentiate each function with respect to  $x$ .**

11)  $y = \sin x^4$

12)  $y = \cos 4x^5$

13)  $y = \cos 3x^3$

14)  $y = \sin 3x^2$

15)  $y = \sec 2x^4$

16)  $y = \tan x^5$

17)  $y = \cot 2x^4$

18)  $y = \csc x^2$

19)  $y = (2x^4 - 5)\sin 3x^3$

$$20) y = \cos 3x^5 \cdot (5x^2 - 1)$$

$$21) y = (-x^5 + 5)\sec x^2$$

$$22) y = \tan 3x^2 \cdot (4x^3 + 5)$$

$$23) y = (-5x^4 - 3)\csc 5x^3$$

$$24) y = \cot 2x^2 \cdot (-x^5 + 5)$$

$$25) y = \sec 3x^3 \cdot (2x^5 - 5)$$

$$26) y = \frac{-5x^5 - 1}{\cos 5x^2}$$

$$27) y = \frac{x^3 - 5}{\tan x^2}$$

$$28) y = \sin \frac{5x^2}{-3x^5 + 1}$$

$$29) y = \sin (\sin 2x^2)$$

$$30) y = \cos (\cos 3x^4)$$

$$31) y = \cos (\sin 2x^4)$$

$$32) y = \sin (\cos 5x^4)$$

$$33) y = \cot (\sec 3x^4)$$

$$34) y = \cot (\cot 4x^2)$$

$$35) y = \tan (\cot 2x^5)$$

$$36) y = \cot (\cot x^2)$$

$$37) y = \sec(\sec 5x^5)$$

$$38) y = \sin(2x^2 - 5)^3$$

$$39) y = \cos(-3x^3 + 4)^2$$

$$40) y = \sec(2x^2 + 1)^3$$

$$41) y = \sqrt{\cot x^3}$$

$$42) y = \cos^2 3x^2$$

$$43) y = \sec^3 2x^2$$

$$44) y = \sin^3 3x^4$$

$$45) y = \cot^2 2x^3$$

$$46) y = \tan(-5x^4 - 4)^2$$

## Limits and Derivatives of Trig Functions

Evaluate each limit.

1)  $\lim_{x \rightarrow -\frac{3\pi}{4}} -\sec(x) \quad \sqrt{2}$

2)  $\lim_{x \rightarrow -\pi} \cos(x) \quad -1$

3)  $\lim_{x \rightarrow \frac{5\pi}{6}} \csc(2x) \quad -\frac{2\sqrt{3}}{3}$

4)  $\lim_{x \rightarrow \frac{3\pi}{4}} -2\sin(2x) \quad 2$

5)  $\lim_{x \rightarrow -\frac{\pi}{2}} -2\sin(x) \quad 2$

6)  $\lim_{x \rightarrow -\frac{\pi}{6}} 2\cos(x) \quad \sqrt{3}$

7)  $\lim_{x \rightarrow \frac{\pi}{3}} -\tan(x) \quad -\sqrt{3}$

8)  $\lim_{x \rightarrow -\pi} \sec(2x) \quad 1$

9)  $\lim_{x \rightarrow \frac{3\pi}{4}} \cot(x) \quad -1$

10)  $\lim_{x \rightarrow \frac{5\pi}{6}} -\sin(2x) \quad \frac{\sqrt{3}}{2}$

Differentiate each function with respect to  $x$ .

11)  $y = \sin x^4$

$$\frac{dy}{dx} = \cos x^4 \cdot 4x^3$$
$$= 4x^3 \cos x^4$$

12)  $y = \cos 4x^5$

$$\frac{dy}{dx} = -\sin 4x^5 \cdot 20x^4$$
$$= -20x^4 \sin 4x^5$$

13)  $y = \cos 3x^3$

$$\frac{dy}{dx} = -\sin 3x^3 \cdot 9x^2$$
$$= -9x^2 \sin 3x^3$$

14)  $y = \sin 3x^2$

$$\frac{dy}{dx} = \cos 3x^2 \cdot 6x$$
$$= 6x \cos 3x^2$$

15)  $y = \sec 2x^4$

$$\frac{dy}{dx} = \sec 2x^4 \tan 2x^4 \cdot 8x^3$$
$$= 8x^3 \sec 2x^4 \tan 2x^4$$

16)  $y = \tan x^5$

$$\frac{dy}{dx} = \sec^2 x^5 \cdot 5x^4$$
$$= 5x^4 \sec^2 x^5$$

17)  $y = \cot 2x^4$

$$\frac{dy}{dx} = -\csc^2 2x^4 \cdot 8x^3$$
$$= -8x^3 \csc^2 2x^4$$

18)  $y = \csc x^2$

$$\frac{dy}{dx} = -\csc x^2 \cot x^2 \cdot 2x$$
$$= -2x \csc x^2 \cot x^2$$

19)  $y = (2x^4 - 5)\sin 3x^3$

$$\frac{dy}{dx} = (2x^4 - 5) \cdot \cos 3x^3 \cdot 9x^2 + \sin 3x^3 \cdot 8x^3$$
$$= x^2(18x^4 \cos 3x^3 - 45 \cos 3x^3 + 8x \sin 3x^3)$$

$$20) y = \cos 3x^5 \cdot (5x^2 - 1)$$

$$\begin{aligned} \frac{dy}{dx} &= \cos 3x^5 \cdot 10x + (5x^2 - 1) \cdot -\sin 3x^5 \cdot 15x^4 \\ &= 5x(2\cos 3x^5 - 15x^5 \sin 3x^5 + 3x^3 \sin 3x^5) \end{aligned}$$

$$21) y = (-x^5 + 5)\sec x^2$$

$$\begin{aligned} \frac{dy}{dx} &= (-x^5 + 5) \cdot \sec x^2 \tan x^2 \cdot 2x + \sec x^2 \cdot -5x^4 \\ &= x \sec x^2 \cdot (-2x^5 \tan x^2 + 10 \tan x^2 - 5x^3) \end{aligned}$$

$$22) y = \tan 3x^2 \cdot (4x^3 + 5)$$

$$\begin{aligned} \frac{dy}{dx} &= \tan 3x^2 \cdot 12x^2 + (4x^3 + 5) \cdot \sec^2 3x^2 \cdot 6x \\ &= 6x(2x \tan 3x^2 + 4x^3 \sec^2 3x^2 + 5 \sec^2 3x^2) \end{aligned}$$

$$23) y = (-5x^4 - 3)\csc 5x^3$$

$$\begin{aligned} \frac{dy}{dx} &= (-5x^4 - 3) \cdot -\csc 5x^3 \cot 5x^3 \cdot 15x^2 + \csc 5x^3 \cdot -20x^3 \\ &= 5x^2 \csc 5x^3 \cdot (15x^4 \cot 5x^3 + 9 \cot 5x^3 - 4x) \end{aligned}$$

$$24) y = \cot 2x^2 \cdot (-x^5 + 5)$$

$$\begin{aligned} \frac{dy}{dx} &= \cot 2x^2 \cdot -5x^4 + (-x^5 + 5) \cdot -\csc^2 2x^2 \cdot 4x \\ &= x(-5x^3 \cot 2x^2 + 4x^5 \csc^2 2x^2 - 20 \csc^2 2x^2) \end{aligned}$$

$$25) y = \sec 3x^3 \cdot (2x^5 - 5)$$

$$\begin{aligned} \frac{dy}{dx} &= \sec 3x^3 \cdot 10x^4 + (2x^5 - 5) \cdot \sec 3x^3 \tan 3x^3 \cdot 9x^2 \\ &= x^2 \sec 3x^3 \cdot (10x^2 + 18x^5 \tan 3x^3 - 45 \tan 3x^3) \end{aligned}$$

$$26) y = \frac{-5x^5 - 1}{\cos 5x^2}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{\cos 5x^2 \cdot -25x^4 - (-5x^5 - 1) \cdot -\sin 5x^2 \cdot 10x}{\cos^2 5x^2} \\ &= \frac{5x(-5x^3 \cos 5x^2 - 10x^5 \sin 5x^2 - 2 \sin 5x^2)}{\cos^2 5x^2} \end{aligned}$$

$$27) y = \frac{x^3 - 5}{\tan x^2}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{\tan x^2 \cdot 3x^2 - (x^3 - 5) \cdot \sec^2 x^2 \cdot 2x}{\tan^2 x^2} \\ &= \frac{x(3x \tan x^2 - 2x^3 \sec^2 x^2 + 10 \sec^2 x^2)}{\tan^2 x^2} \end{aligned}$$

$$28) y = \sin \frac{5x^2}{-3x^5 + 1}$$

$$\frac{dy}{dx} = \cos \frac{5x^2}{-3x^5 + 1} \cdot \frac{(-3x^5 + 1) \cdot 10x - 5x^2 \cdot -15x^4}{(-3x^5 + 1)^2}$$

$$= \frac{5x \cos \frac{5x^2}{-3x^5 + 1} \cdot (9x^5 + 2)}{(-3x^5 + 1)^2}$$

$$29) y = \sin(\sin 2x^2)$$

$$\frac{dy}{dx} = \cos(\sin 2x^2) \cdot \cos 2x^2 \cdot 4x$$

$$= 4x \cos(\sin 2x^2) \cos 2x^2$$

$$30) y = \cos(\cos 3x^4)$$

$$\frac{dy}{dx} = -\sin(\cos 3x^4) \cdot -\sin 3x^4 \cdot 12x^3$$

$$= 12x^3 \sin(\cos 3x^4) \sin 3x^4$$

$$31) y = \cos(\sin 2x^4)$$

$$\frac{dy}{dx} = -\sin(\sin 2x^4) \cdot \cos 2x^4 \cdot 8x^3$$

$$= -8x^3 \sin(\sin 2x^4) \cos 2x^4$$

$$32) y = \sin(\cos 5x^4)$$

$$\frac{dy}{dx} = \cos(\cos 5x^4) \cdot -\sin 5x^4 \cdot 20x^3$$

$$= -20x^3 \cos(\cos 5x^4) \sin 5x^4$$

$$33) y = \cot(\sec 3x^4)$$

$$\frac{dy}{dx} = -\csc^2(\sec 3x^4) \cdot \sec 3x^4 \tan 3x^4 \cdot 12x^3$$

$$= -12x^3 \csc^2(\sec 3x^4) \sec 3x^4 \tan 3x^4$$

$$34) y = \cot(\cot 4x^2)$$

$$\frac{dy}{dx} = -\csc^2(\cot 4x^2) \cdot -\csc^2 4x^2 \cdot 8x$$

$$= 8x \csc^2(\cot 4x^2) \csc^2 4x^2$$

$$35) y = \tan(\cot 2x^5)$$

$$\frac{dy}{dx} = \sec^2(\cot 2x^5) \cdot -\csc^2 2x^5 \cdot 10x^4$$

$$= -10x^4 \sec^2(\cot 2x^5) \csc^2 2x^5$$

$$36) y = \cot(\cot x^2)$$

$$\frac{dy}{dx} = -\csc^2(\cot x^2) \cdot -\csc^2 x^2 \cdot 2x$$

$$= 2x \csc^2(\cot x^2) \csc^2 x^2$$

$$37) y = \sec(\sec 5x^5)$$

$$\begin{aligned}\frac{dy}{dx} &= \sec(\sec 5x^5) \tan(\sec 5x^5) \cdot \sec 5x^5 \tan 5x^5 \cdot 25x^4 \\ &= 25x^4 \sec(\sec 5x^5) \tan(\sec 5x^5) \sec 5x^5 \tan 5x^5\end{aligned}$$

$$38) y = \sin(2x^2 - 5)^3$$

$$\begin{aligned}\frac{dy}{dx} &= \cos(2x^2 - 5)^3 \cdot 3(2x^2 - 5)^2 \cdot 4x \\ &= 12x \cos(2x^2 - 5)^3 \cdot (2x^2 - 5)^2\end{aligned}$$

$$39) y = \cos(-3x^3 + 4)^2$$

$$\begin{aligned}\frac{dy}{dx} &= -\sin(-3x^3 + 4)^2 \cdot 2(-3x^3 + 4) \cdot -9x^2 \\ &= 18x^2 \sin(-3x^3 + 4)^2 \cdot (-3x^3 + 4)\end{aligned}$$

$$40) y = \sec(2x^2 + 1)^3$$

$$\begin{aligned}\frac{dy}{dx} &= \sec(2x^2 + 1)^3 \tan(2x^2 + 1)^3 \cdot 3(2x^2 + 1)^2 \cdot 4x \\ &= 12x \sec(2x^2 + 1)^3 \tan(2x^2 + 1)^3 \cdot (2x^2 + 1)^2\end{aligned}$$

$$41) y = \sqrt{\cot x^3}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2} \cdot (\cot x^3)^{-\frac{1}{2}} \cdot -\csc^2 x^3 \cdot 3x^2 \\ &= -\frac{3x^2 \csc^2 x^3}{2(\cot x^3)^{\frac{1}{2}}}\end{aligned}$$

$$42) y = \cos^2 3x^2$$

$$\begin{aligned}\frac{dy}{dx} &= 2\cos 3x^2 \cdot -\sin 3x^2 \cdot 6x \\ &= -12x \cos 3x^2 \sin 3x^2\end{aligned}$$

$$43) y = \sec^3 2x^2$$

$$\begin{aligned}\frac{dy}{dx} &= 3\sec^2 2x^2 \cdot \sec 2x^2 \tan 2x^2 \cdot 4x \\ &= 12x \sec^3 2x^2 \tan 2x^2\end{aligned}$$

$$44) y = \sin^3 3x^4$$

$$\begin{aligned}\frac{dy}{dx} &= 3\sin^2 3x^4 \cdot \cos 3x^4 \cdot 12x^3 \\ &= 36x^3 \sin^2 3x^4 \cos 3x^4\end{aligned}$$

$$45) y = \cot^2 2x^3$$

$$\begin{aligned}\frac{dy}{dx} &= 2\cot 2x^3 \cdot -\csc^2 2x^3 \cdot 6x^2 \\ &= -12x^2 \cot 2x^3 \csc^2 2x^3\end{aligned}$$

$$46) y = \tan(-5x^4 - 4)^2$$

$$\begin{aligned}\frac{dy}{dx} &= \sec^2(-5x^4 - 4)^2 \cdot 2(-5x^4 - 4) \cdot -20x^3 \\ &= -40x^3 \sec^2(-5x^4 - 4)^2 (-5x^4 - 4)\end{aligned}$$