Calculus Name________________

Cross Sections - y-axis

For each problem, find the volume of the specified solid.

1) The base of a solid is the region enclosed by the semicircle $y = \sqrt{16 - x^2}$ and the x-axis. Cross-sections perpendicular to the y-axis are squares.

$$\int_0^4 \left(\sqrt{16 - y^2} + \sqrt{16 - y^2}\right)^2 dy$$

$$= \frac{512}{3} \approx 170.667$$

2) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 49$. Cross-sections perpendicular to the y-axis are squares.

$$\int_{-7}^{7} \left(\sqrt{49 - y^2} + \sqrt{49 - y^2}\right)^2 dy$$

$$= \frac{5488}{3} \approx 1829.333$$

3) The base of a solid is the region enclosed by the semicircle $y = \sqrt{16 - x^2}$ and the x-axis. Cross-sections perpendicular to the y-axis are rectangles with heights twice that of the side in the xy-plane.

$$2\int_0^4 \left(\sqrt{16 - y^2} + \sqrt{16 - y^2}\right)^2 dy$$

$$= \frac{1024}{3} \approx 341.333$$

4) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 25$. Cross-sections perpendicular to the y-axis are rectangles with heights half that of the side in the xy-plane.

$$\frac{1}{2}\int_{-5}^{5} \left(\sqrt{25 - y^2} + \sqrt{25 - y^2}\right)^2 dy$$

$$= \frac{1000}{3} \approx 333.333$$

5) The base of a solid is the region enclosed by $y = -\frac{x^2}{9} + 4$ and $y = 0$. Cross-sections perpendicular to the y-axis are semicircles.

$$\frac{\pi}{8}\int_0^4 \left(\sqrt{36 - 9y} + \sqrt{36 - 9y}\right)^2 dy$$

$$= 36\pi \approx 113.097$$

6) The base of a solid is the region enclosed by $y = -\frac{x^2}{4} + 1$ and $y = 0$. Cross-sections perpendicular to the y-axis are isosceles right triangles with one leg in the xy-plane.

$$\frac{1}{2}\int_0^1 \left(\sqrt{4-4y} + \sqrt{4-4y}\right)^2 dy$$
$$= 4$$

7) The base of a solid is the region enclosed by $y = -x^2 + 4$ and $y = 0$. Cross-sections perpendicular to the y-axis are isosceles right triangles with the hypotenuse in the base.

$$\frac{1}{4}\int_0^4 \left(\sqrt{4-y} + \sqrt{4-y}\right)^2 dy$$
$$= 8$$

8) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 36$. Cross-sections perpendicular to the y-axis are equilateral triangles.

$$\frac{\sqrt{3}}{4}\int_{-6}^{6} \left(\sqrt{36-y^2} + \sqrt{36-y^2}\right)^2 dy$$
$$= 288\sqrt{3} \approx 498.831$$

9) The base of a solid is the region enclosed by a circle with a diameter of 8. Cross-sections perpendicular to the y-axis are equilateral triangles.

$$\frac{\sqrt{3}}{4}\int_{-4}^{4} \left(\sqrt{16-y^2} + \sqrt{16-y^2}\right)^2 dy$$
$$= \frac{256\sqrt{3}}{3} \approx 147.802$$

10) The base of a solid is the region enclosed by a semicircle with a radius of 3, lying flat on the x-axis. Cross-sections perpendicular to the y-axis are equilateral triangles.

$$\frac{\sqrt{3}}{4}\int_0^3 \left(\sqrt{9-y^2} + \sqrt{9-y^2}\right)^2 dy$$
$$= 18\sqrt{3} \approx 31.177$$

