Calculus Name________________________________

Volume of Solids with Known Cross Sections

For each problem, find the volume of the specified solid.

1) The base of a solid is the region enclosed by $y = -\frac{x^2}{4} + 1$ and $y = 0$. Cross-sections perpendicular to the x-axis are squares.

$$\int_{-2}^{2} \left(-\frac{x^2}{4} + 1\right)^2 dx$$

$$= \frac{32}{15} \approx 2.133$$

2) The base of a solid is the region enclosed by $y = -\frac{x^2}{4} + 1$ and $y = 0$. Cross-sections perpendicular to the x-axis are rectangles with heights half that of the side in the xy-plane.

$$\frac{1}{2}\int_{-2}^{2} \left(-\frac{x^2}{4} + 1\right)^2 dx$$

$$= \frac{16}{15} \approx 1.067$$

3) The base of a solid is the region enclosed by $y = -\frac{x^2}{4} + 1$ and $y = 0$. Cross-sections perpendicular to the x-axis are semicircles.

$$\frac{\pi}{8}\int_{-2}^{2} \left(-\frac{x^2}{4} + 1\right)^2 dx$$

$$= \frac{4\pi}{15} \approx 0.838$$

4) The base of a solid is the region enclosed by the semicircle $y = \sqrt{25 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are squares.

$$\int_{-5}^{5} \left(\sqrt{25 - x^2}\right)^2 dx$$

$$= \frac{500}{3} \approx 166.667$$

5) The base of a solid is the region enclosed by the semicircle $y = \sqrt{25 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are rectangles with heights half that of the side in the x y-plane.

$$\frac{1}{2}\int_{-5}^{5} \left(\sqrt{25 - x^2}\right)^2 dx$$

$$= \frac{250}{3} \approx 83.333$$

6) The base of a solid is the region enclosed by the semicircle $y = \sqrt{25 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are semicircles.

$$\frac{\pi}{8}\int_{-5}^{5}\left(\sqrt{25 - x^2}\right)^2 dx$$

$$= \frac{125\pi}{6} \approx 65.45$$

7) The base of a solid is the region enclosed by the semicircle $y = \sqrt{49 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.

$$\frac{1}{4}\int_{-7}^{7}\left(\sqrt{49 - x^2}\right)^2 dx$$

$$= \frac{343}{3} \approx 114.333$$

8) The base of a solid is the region enclosed by $y = 4$ and $y = x^2$. Cross-sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.

$$\frac{1}{4}\int_{-2}^{2}\left(4 - x^2\right)^2 dx$$

$$= \frac{128}{15} \approx 8.533$$

9) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{49} + \frac{y^2}{4} = 1$. Cross-sections perpendicular to the x-axis are isosceles right triangles with one leg in the xy-plane.

$$\frac{1}{2}\int_{-7}^{7}\left(\sqrt{4 - \frac{4x^2}{49}} + \sqrt{4 - \frac{4x^2}{49}}\right)^2 dx$$

$$= \frac{224}{3} \approx 74.667$$

10) The base of a solid is the region enclosed by the semicircle $y = \sqrt{16 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are isosceles right triangles with one leg in the xy-plane.

$$\frac{1}{2}\int_{-4}^{4}\left(\sqrt{16 - x^2}\right)^2 dx$$

$$= \frac{128}{3} \approx 42.667$$

11) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 25$. Cross-sections perpendicular to the x-axis are equilateral triangles.

$$\frac{\sqrt{3}}{4}\int_{-5}^{5}\left(\sqrt{25 - x^2} + \sqrt{25 - x^2}\right)^2 dx$$

$$= \frac{500\sqrt{3}}{3} \approx 288.675$$

12) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. Cross-sections perpendicular to the x-axis are equilateral triangles.

$$\frac{\sqrt{3}}{4}\int_{-2}^{2}\left(\sqrt{9 - \frac{9x^2}{4}} + \sqrt{9 - \frac{9x^2}{4}}\right)^2 dx$$
$$= 24\sqrt{3} \approx 41.569$$

