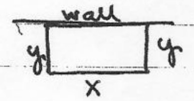
1. a. The area of the rectangular enclosure is being maximized.

$A = LW$

b. There is 120m fencing for three walls.



$x + 2y = 120$

c. $x + 2y = 120$

$x = 120 - 2y$

d. $A = LW$

$A = (x)(y)$

$A = (120 - 2y)(y)$

$A = 120y - 2y^2$

e. $A' = 120 - 4y = 0$

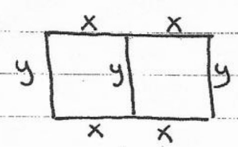
$y = 30\text{m} \quad x = 120 - 2(30) = 60\text{m}$

f. Since $x = 60\text{m}$ and $y = 30\text{m}$, the maximum area of the enclosure is 1800m^2

2. Maximize: area of the enclosure

$A = LW$

Constraints: There is 102m fencing for the enclosure.



$4x + 3y = 102$

Because $4x + 3y = 102$

$y = 34 - \frac{4}{3}x$

$A = LW$

$A = (2x)(y)$

$A = (2x)(34 - \frac{4}{3}x)$

$A = 68x - \frac{8}{3}x^2$

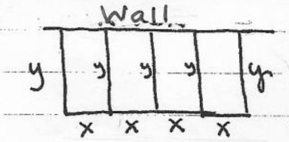
$A' = 68 - \frac{16}{3}x = 0$

$x = \frac{51}{4} \approx 12.75\text{m}$

$y = 17$

The maximum area is $2(12.75)(17) = 433.5\,\text{m}^2$

3, Maximize: Area of 4 pens
Constraint: There is 150 feet of fencing.



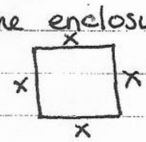
Area $= 4xy$
Constraint $= 4x + 5y = 150$
$y = 30 - \frac{4}{5}x$

Area $= 4xy$
$A = 4x(30 - \frac{4}{5}x)$
$A = 120x - \frac{16}{5}x^2$
$A' = 120 - \frac{32}{5}x = 0$
$x = 18.75$ ft. $\quad y = 30 - \frac{4}{5}(18.75) = 15$ ft.

If $x = 18.75$ ft. and $y = 15$ ft, the maximum area of all four pens is 1125 ft^2.

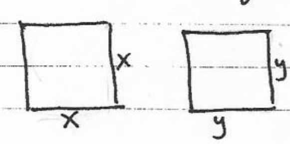
4, Minimize: Area of the square enclosure(s) AND Maximize the area.
Constraint: There is 200 m of fencing to make either one enclosure or two separate square enclosures.

One enclosure



$4x = 200$
$x = 50$ m
Therefore
$\boxed{A = 50^2 = 2500 \text{ m}^2}$

Two separate square enclosures:

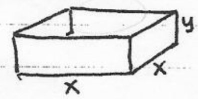


Total Area $= x^2 + y^2$
Constraint: $4x + 4y = 200$

$4x + 4y = 200$
$y = 50 - x$
$A = x^2 + y^2$
$A = x^2 + (50 - x)^2$
$A = x^2 + (2500 - 100x + x^2)$
$A = 2x^2 - 100x + 2500$
$A' = 4x - 100 = 0$
$x = 25$ m; $y = 50 - 25 = 25$ m
$\boxed{\text{Total Area} = 25^2 + 25^2 = 1250 \text{ m}^2}$

Max. Area $= 2500$ m^2 using one enclosure. Min. Area $= 1250$ m^2 using two enclosures.

5. Maximize: Volume of open box with square base.
Constraint: Surface area $= 108 \text{ in}^2$.



Max: $V = x^2 y$

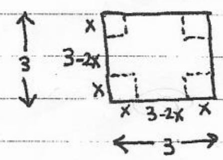
Constraint: $x^2 + 4xy = 108$

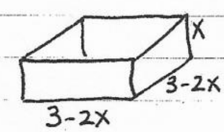
$$y = \frac{27}{x} - \frac{1}{4}x$$

$$V = x^2 y$$
$$V = x^2\left(\frac{27}{x} - \frac{1}{4}x\right)$$
$$V = 27x - \frac{1}{4}x^3$$
$$V' = 27 - \frac{3}{4}x^2 = 0$$
$$x = 6'' \quad y = \frac{27}{x} - \frac{1}{4}x = 3''$$

$\boxed{\text{The dimensions of the open box are } 6'' \times 6'' \times 3''.}$

6. Maximize: Volume of open box
Constraint: Must use square piece of cardboard: $3' \times 3'$.





$$V = LWH$$
$$V = (3-2x)(3-2x)(x)$$
$$V = (9 - 12x + 4x^2)(x)$$
$$V = 4x^3 - 12x^2 + 9x$$

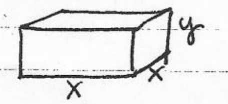
$$V' = 12x^2 - 24x + 9 = 0$$
$$= 3(4x^2 - 8x + 3) = 0$$
$$= 3(2x-3)(2x-1) = 0$$
$$x = \frac{3}{2} \text{ or } \frac{1}{2}$$

x cannot $= \frac{3}{2}$, because of the constraint: Side of box $= 3-2x$

$3 - 2\left(\frac{3}{2}\right) = 0$! There would be no box if $x = \frac{3}{2}$!

$\boxed{\text{So, } x = \tfrac{1}{2} \text{ and the maximum volume is } 2 \text{ ft}^3.}$

7. Maximize: Volume of closed box with square base
Constraint: Surface Area of box is 100 in^2.



$V = LWH$

$V = x^2y$

Surface Area $= 2x^2 + 4xy = 100$

$y = \frac{25}{x} - \frac{1}{2}x$

$V = x^2y$

$V = x^2\left(\frac{25}{x} - \frac{1}{2}x\right)$

$V = 25x - \frac{1}{2}x^3$

$V' = 25 - \frac{3}{2}x^2 = 0$

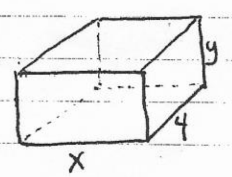
$x = \sqrt{\frac{50}{3}} \approx 4.08$ in.

$y = \frac{25}{x} - \frac{1}{2}x$

$y \approx 4.08$ in.

$\boxed{\text{The maximum volume of the box is about } 68.04 \text{ in}^3}$

8. Minimize: Cost of open rectangular tank.
Constraint: Base cost= \$10/m^2 and sides cost= \$5/m^2
Width = 4m
Volume = 36 m^3



$V = 4xy = 36$

$y = \frac{9}{x}$ substitute here

Base, 2 sides, 2 sides

$SA = 4x + (4y)(2) + (xy)(2)$

$SA = 4x + 8y + 2xy$

Cost $= (4x)(\$10) + (8y)(\$5) + (2xy)(\$5)$

$C = (4x)(10) + (8)\left(\frac{9}{x}\right)(5) + \left(2x\left(\frac{9}{x}\right)\right)(5)$

$C = 40x + 360x^{-1} + 90$

$C' = 40 - 360x^{-2}$

$C' = 40 - \frac{360}{x^2} = 0$

$C' = \frac{40x^2 - 360}{x^2} = 0$

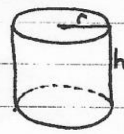
$40x^2 - 360 = 0$

$x = 3$

$y = \frac{9}{x} = 3$

$\boxed{\text{Dimensions are } 3\text{m} \times 3\text{m} \times 4 \text{ Cost is } \$330.}$

9. Minimize: Surface area of cylindrical container.
Constraint: Volume = 355 ml Note: $1 cm^3 = 1 ml$



Minimize $SA = \underbrace{2\pi r^2}_{\text{top \& bottom}} + \underbrace{2\pi r h}_{\text{sides}}$

Constraint $V = 355 = \pi r^2 h$

$$h = \frac{355}{\pi r^2}$$

$$SA = 2\pi r^2 + 2\pi r h$$

$$SA = 2\pi r^2 + 2\pi r\left(\frac{355}{\pi r^2}\right)$$

$$SA = 2\pi r^2 + \frac{710}{r}$$

$$SA = 2\pi r^2 + 710 r^{-1}$$

$$SA' = 4\pi r - 710 r^{-2}$$

$$SA' = 4\pi r - \frac{710}{r^2} = 0$$

$$SA' = 4\pi r = \frac{710}{r^2}$$

$$r = \sqrt[3]{\frac{710}{4\pi}} \approx 3.84 cm \qquad h = \frac{355}{\pi r^2} \approx 7.66 cm$$

The dimensions are radius = 3.84cm and height = 7.66 cm.

10. Minimize Cost of Surface Area of can : $SA = 2\pi r^2 + 2\pi r h$
Constraint: Top & bottom cost \$2/in²; sides cost \$6/in².
$V = 300 in^3$

$$V = \pi r^2 h = 300 \qquad Cost = 2\pi r^2(\$2) + 2\pi r h(\$6)$$

$$h = \frac{300}{\pi r^2} \qquad C = 4\pi r^2 + 12\pi r\left(\frac{300}{\pi r^2}\right)$$

$$C = 4\pi r^2 + 3600 r^{-1}$$

$$C' = 8\pi r - \frac{3600}{r^2} = 0$$

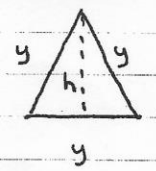
$$r = \sqrt[3]{\frac{3600}{8\pi}} \approx 5.23 in$$

$$h \approx 3.49 in.$$

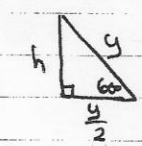
Dimensions are radius = 5.23 in.
height = 3.49 in.
Cost ≈ \$1032

11. Maximize Area of triangle and square : $A = \frac{1}{2}bh + x^2$

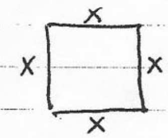
Constraint: Combined perimeter is 10 : $P = 3y + 4x = 10$



← Equilateral triangle:



$h = \frac{y\sqrt{3}}{2}$



$P = 3y + 4x = 10$

$x = \frac{10-3y}{4}$

$$\text{Area} = \frac{1}{2}bh + x^2$$

$$= \frac{1}{2}y\left(\frac{y\sqrt{3}}{2}\right) + \left(\frac{10-3y}{4}\right)^2$$

$$A = \frac{y^2\sqrt{3}}{4} + \frac{100-60y+9y^2}{16}$$

$$A = \frac{y^2\sqrt{3}}{4} + \frac{25}{4} - \frac{15}{4}y + \frac{9}{16}y^2$$

$$A' = \frac{\sqrt{3}}{2}y - \frac{15}{4} + \frac{9}{8}y = 0$$

$y \approx 1.883$ $x \approx 1.088$

The triangle has sides = 1.883.
The square has sides = 1.088.

12. Maximize Area of circle and square: $A = \pi r^2 + x^2$

Constraint: Perimeter of both = 16.

$$P = 2\pi r + 4x = 16$$

$$r = \frac{8-2x}{\pi}$$

$$A = \pi r^2 + x^2$$

$$A = \pi\left(\frac{8-2x}{\pi}\right)^2 + x^2$$

$$A = x^2 + \frac{64}{\pi} - \frac{32x}{\pi} + \frac{4x^2}{\pi}$$

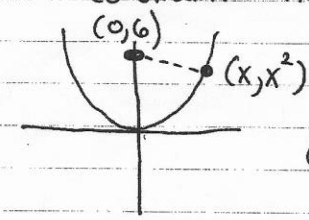
$$A' = 2x - \frac{32}{\pi} + \frac{8}{\pi}x = 0$$

$$x \approx 2.24 \qquad r \approx 1.12$$

$$\boxed{\begin{array}{l}\text{The sides of the square are } 2.24.\\ \text{The radius of the circle is } 1.12.\end{array}}$$

13. Minimize distance from $(0,6)$ to $y = x^2$.

Constraint: the point $(0,6)$ and $y = x^2$.



$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Our two points are $(0,6)$ and (x, x^2)

$$D = \sqrt{(x-0)^2 + (x^2-6)^2}$$

$$D = (x^2 + x^4 - 12x^2 + 36)^{1/2}$$

$$D = (x^4 - 11x^2 + 36)^{1/2}$$

$$D' = \tfrac{1}{2}(x^4 - 11x^2 + 36)^{-1/2}(4x^3 - 22x)$$

$$D' = \frac{4x^3 - 22x}{2\sqrt{x^4 - 11x^2 + 36}} = 0 \rightarrow 4x^3 - 22x = 0$$

$$x = 0, \sqrt{11/2}$$

If $x = 0$, $y = 0$ and $D = 6$

$\boxed{\text{If } x = \sqrt{\tfrac{11}{2}},\ y = \tfrac{11}{2} \text{ and } D = 5.75}$ minimum distance