Calculus Name Solutions

# Limits - Algebraically

**Evaluate each limit.** #1-6: Functions are continuous at x=a. Substitute

1) $\lim_{x\to -1}(x-2) = -1-2 = \boxed{-3}$

2) $\lim_{x\to 0}\sqrt{-2x+3} = \sqrt{-2(0)+3} = \boxed{\sqrt{3}}$

3) $\lim_{x\to -1}(x^3-3x^2) = (-1)^3 - 3(-1)^2 = \boxed{-4}$

4) $\lim_{x\to -3} -\dfrac{3x}{x+6} = \dfrac{-3(-3)}{-3+6} = \boxed{3}$

5) $\lim_{x\to -\frac{\pi}{6}} \cos(x) = \cos\left(-\frac{\pi}{6}\right) = \boxed{\frac{\sqrt{3}}{2}}$

6) $\lim_{x\to 0}\sec(x) = \dfrac{1}{\cos 0} = \boxed{1}$

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#7-20: Rational functions may have Point Discontinuities (Holes) and/or Essential (Infinite) Discontinuities. Determine if the hole affects your limit as x→a (limit = constant) or the Vertical Asymptote affects your limit as x→a (limit = +∞ or -∞ or DNE)

7) $\lim_{x\to 3}\dfrac{x-3}{x^2-4x+3} = \lim_{x\to 3}\dfrac{x-3}{(x-3)(x-1)} = \lim_{x\to 3}\dfrac{1}{x-1} = \dfrac{1}{3-1} = \boxed{\frac{1}{2}}$

Hole at x=3.

Factor/cancel/substitute

8) $\lim_{x\to 2}\dfrac{x^2-3x+2}{x-2} = \lim_{x\to 2}\dfrac{(x-1)(x-2)}{x-2} = \lim_{x\to 2}(x-1) = 2-1 = \boxed{1}$

Hole at x=2

Factor/cancel/substitute

9) $\lim_{x\to -1} -\dfrac{x+1}{x^2+3x+2} = \lim_{x\to -1}\dfrac{-(x+1)}{(x+2)(x+1)} = \lim_{x\to -1}\dfrac{-1}{x+2} = \dfrac{-1}{-1+2} = \boxed{-1}$

Hole at x=-1.

Factor/cancel/substitute

10) $\lim_{x\to 1}\dfrac{x^2-5x+4}{x-1} = \lim_{x\to 1}\dfrac{(x-1)(x-4)}{x-1} = \lim_{x\to 1}(x-4) = 1-4 = \boxed{-3}$

Hole at x=1

Factor/cancel/substitute

11) $\lim_{x\to -1}\dfrac{x+2}{x^2+2x+1} = \lim_{x\to -1}\dfrac{x+2}{(x+1)^2}$

VA at x = -1.

Find LHL and RHL:

LHL: $\dfrac{+\infty}{+\infty} = \infty$ (-1.1)

RHL: $\dfrac{+\infty}{+\infty} = \infty$ (-.9)

$\boxed{\infty}$

12) $\lim_{x\to -2}\dfrac{x-2}{x^2-4} = \lim_{x\to -2}\dfrac{x-2}{(x+2)(x-2)} = \lim_{x\to -2}\dfrac{1}{x+2}$

Hole at x=2 (N/A)

VA at x=-2,

find LHL and RHL

LHL: $\dfrac{+\infty}{-\infty} = -\infty$ (-2.1)

RHL: $\dfrac{+\infty}{+\infty} = \infty$ (-1.9)

$\boxed{\text{DNE}}$

13) $\lim_{x\to -2}\dfrac{x^2}{2x+4} = \lim_{x\to -2}\dfrac{x^2}{2(x+2)}$

VA at x = -2

Find LHL, RHL

LHL: $\dfrac{+\infty}{-\infty} = -\infty$ (-2.1)

RHL: $\dfrac{+\infty}{+\infty} = \infty$ (-1.9)

$\boxed{\text{DNE}}$

14) $\lim_{x\to 2}\dfrac{x-1}{x^2-4x+4} = \lim_{x\to 2}\dfrac{x-1}{(x-2)^2}$

VA at x = 2

find LHL, RHL

LHL: $\dfrac{+\infty}{+\infty} = \infty$ (1.9)

RHL: $\dfrac{+\infty}{+\infty} = \infty$ (2.1)

$\boxed{\infty}$

15) $\lim_{x\to 1}\dfrac{x-3}{x^2-2x+1} = \lim_{x\to 1}\dfrac{x-3}{(x-1)^2}$

VA at x=1

Find LHL, RHL

LHL: $\dfrac{-\infty}{+\infty} = -\infty$ (.9)

RHL: $\dfrac{-\infty}{+\infty} = -\infty$ (1.1)

$\boxed{-\infty}$

16) $\lim_{x\to 1}\dfrac{1}{x^2-1} = \lim_{x\to 1}\dfrac{1}{(x+1)(x-1)}$

VA at x=-1 (N/A) and x=1

Find LHL, RHL

LHL: $\dfrac{1}{-\infty}$ (.9)

RHL: $\dfrac{1}{+\infty}$ (1.1)

$\boxed{\text{DNE}}$

17) $\lim_{x\to 3} \frac{x-3}{x^2-2x-3} = \lim_{x\to 3} \frac{x-3}{(x-3)(x+1)} = \lim_{x\to 3} \frac{1}{x+1}$

$= \frac{1}{3+1} = \boxed{\frac{1}{4}}$

VA at $x=-1$: N/A

Hole at $x=3$.

Factor/cancel/substitute

18) $\lim_{x\to 1} -\frac{x-1}{x^2-4x+3} = \lim_{x\to 1} \frac{-(x-1)}{(x-1)(x-3)} = \lim_{x\to 1} \frac{-1}{x-3}$

$= \frac{-1}{1-3} = \boxed{\frac{1}{2}}$

VA at $x=3$: N/A

Hole at $x=1$.

Factor/cancel/substitute

19) $\lim_{x\to -3} -\frac{x+3}{x^2+5x+6} = \lim_{x\to -3} \frac{-(x+3)}{(x+3)(x+2)}$

$= \lim_{x\to -3} \frac{-1}{x+2}$

$= \frac{-1}{-3+2} = \boxed{1}$

VA at $x=-2$: N/A

Hole at $x=-3$.

Factor/cancel/substitute

20) $\lim_{x\to 2} -\frac{x-2}{x^2-4} = \lim_{x\to 2} \frac{-(x-2)}{(x-2)(x+2)} = \lim_{x\to 2} \frac{-1}{x+2}$

$= \frac{-1}{2+2} = \boxed{-\frac{1}{4}}$

VA at $x=-2$: N/A

Hole at $x=2$.

Factor/cancel/substitute

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21) $\lim_{x\to 0} \left(\frac{\sqrt{x+1}-1}{x}\right)\left(\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right)$

$= \lim_{x\to 0} \left(\frac{x+1-1}{x(\sqrt{x+1}+1)}\right) = \lim_{x\to 0} \left(\frac{x}{x(\sqrt{x+1}+1)}\right)$

$= \lim_{x\to 0} \frac{1}{\sqrt{x+1}+1} = \frac{1}{\sqrt{0+1}+1} = \boxed{\frac{1}{2}}$

22) $\lim_{x\to 0} \left(\frac{x}{\sqrt{x+3}-\sqrt{3}}\right)\left(\frac{\sqrt{x+3}+\sqrt{3}}{\sqrt{x+3}+\sqrt{3}}\right)$

$= \lim_{x\to 0} \left(\frac{x(\sqrt{x+3}+\sqrt{3})}{x+3-3}\right) = \lim_{x\to 0} \frac{x(\sqrt{x+3}+\sqrt{3})}{x}$

$= \lim_{x\to 0} (\sqrt{x+3}+\sqrt{3}) = \sqrt{0+3}+\sqrt{3} = \boxed{2\sqrt{3}}$

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23) $\lim_{x\to 1} f(x),\ f(x) = \begin{cases} -x-4, & x \le 1 \\ x-2, & x > 1 \end{cases}$ Note the conditional is an inequality

At the border $x=1$, find the LHL and RHL

LHL: $-(1)-4=-5$

RHL: $1-2=-1$

$\boxed{\text{DNE}}$

24) $\lim_{x\to 0} f(x),\ f(x) = \begin{cases} -x-2, & x < 0 \\ -x^2, & x \ge 0 \end{cases}$

LHL: $-0-2=-2$

RHL: $-(0)^2=0$

$\boxed{\text{DNE}}$

25) $\lim_{x\to 1} f(x),\ f(x) = \begin{cases} x^2+2x, & x \ne 1 \\ 0, & x = 1 \end{cases}$

This is a quadratic everywhere except at $x=1$. There is a hole at $x=1$. (a removable discontinuity)

The LHL = RHL

$(1)^2+2(1) = \boxed{3}$

26) $\lim_{x\to -2} f(x),\ f(x) = \begin{cases} 1+\frac{x}{2}, & x \ne -2 \\ -3, & x = -2 \end{cases}$

This is a linear function everywhere except at $x=-2$.

The LHL = RHL

$1 + \frac{-2}{2} = \boxed{0}$

27) $\lim_{x\to 0} f(x),\ f(x) = \begin{cases} -x-3, & x < 0 \\ -x^2-1, & x \ge 0 \end{cases}$

The conditional statement is an inequality so the left side and right side are two different functions.

LHL: $-0-3=-3$

RHL: $-(0)^2-1=-1$

$\boxed{\text{DNE}}$

28) $\lim_{x\to -1} f(x),\ f(x) = \begin{cases} -2x+2, & x \ne -1 \\ -1, & x = -1 \end{cases}$

The conditional statement is an equality

This is a linear function everywhere except at $x=-1$.

The LHL = RHL

$-2(-1)+2 = \boxed{4}$

End Behavior! $x \to \infty$ or $x \to -\infty$

29) $\lim_{x\to\infty} (-x^4 + 3x^2 + 3x)$

A negative even degree

$\boxed{-\infty}$

30) $\lim_{x\to-\infty} (x^5 - 2x^3 - 2)$

A positive odd degree

$\boxed{-\infty}$

31) $\lim_{x\to-\infty} -e^x = \boxed{0}$

An exponential growth function reflected over the horizontal asymptote.

32) $\lim_{x\to\infty} (e^{-4x} - 4) = \boxed{-4}$

An exponential growth function reflected over the y-axis.

33) $\lim_{x\to-\infty} -\frac{x^2}{x^2-9} = \boxed{-1}$

Rational function. Same degree numerator and denominator. limit = Horizontal Asymptote

34) $\lim_{x\to-\infty} \frac{3x}{x-2} = \boxed{3}$

same as 33.

35) $\lim_{x\to-\infty} \frac{4x}{x^2+4} = \boxed{0}$

Rational function. Higher degree in denominator limit = Horizontal asymptote = 0.

36) $\lim_{x\to\infty} \frac{15}{x^2+3} = \boxed{0}$

Same as 35.

37) $\lim_{x\to-\infty} \frac{x^2}{2x-4} = \boxed{-\infty}$

Rational function. Higher degree in numerator. No horizontal asymptotes, limit = $+\infty$ or $-\infty$. Substitute and note signs: $\frac{(-\infty)^2}{2(-\infty)-4} = -\infty$

38) $\lim_{x\to\infty} \frac{x^3}{2x^2-1} = \boxed{\infty}$

No Horizontal Asymptote. Substitute

$\frac{(\infty)^3}{2(\infty)^2-1} = \frac{+}{+} = \boxed{\infty}$

39) $\lim_{x\to\infty} \frac{x^4}{3x^2-3} = \boxed{\infty}$

No HA. Substitute. $\frac{(\infty)^4}{3(\infty)^2-3} = \frac{+}{+} = \boxed{\infty}$

40) $\lim_{x\to-\infty} \frac{3x^4}{4x^2-1} = \boxed{\infty}$

No HA. Substitute $\frac{3(-\infty)^4}{4(-\infty)^2-1} = \frac{+}{+} = \boxed{\infty}$

41) $\lim_{x\to\infty} -\frac{3x^4}{2x^2+1} = \boxed{-\infty}$

No HA. Substitute. $\frac{-3(\infty)^4}{2(\infty)^2+1} = \frac{-}{+} = \boxed{-\infty}$

42) $\lim_{x\to-\infty} -\frac{2x^3}{3x^2-3} = \boxed{\infty}$

No HA. Substitute. $\frac{-2(-\infty)^3}{3(-\infty)^2-3} = \frac{+}{+} = \boxed{\infty}$

43) $\lim_{x\to\infty} \frac{2x-3}{\sqrt{x^2+3}} = \boxed{2}$

44) $\lim_{x\to-\infty} \frac{3x+2}{\sqrt{x^2+4}} = \boxed{-3}$

45) $\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{x+2} = \boxed{-1}$

46) $\lim_{x\to\infty} \sqrt[3]{\frac{3x^2+1}{x^2+1}} = \boxed{\sqrt[3]{3}}$