Area Between Curves

For each problem, find the area of the region enclosed by the curves.

1) $y = x^2 - 2x,\ y = -x^2 + 4,$
$x = -1,\ x = 3$

2) $y = \frac{x^2}{2} + 2x - 1,\ y = -\frac{x^2}{2} - 2x + 4,$
$x = -2,\ x = 2$

3) $y = -\cos x,\ y = \cos x,$
$x = -\pi,\ x = \frac{\pi}{2}$

4) $y = -2\sin x,\ y = 2\sin x,$
$x = -\frac{\pi}{2},\ x = \frac{3\pi}{4}$

5) $x = -y^2 + 2y + 1,\ x = y^2 - 2y - 5,$
$y = -2,\ y = 3$

6) $x = -y^2 + 8y - 12,\ x = -y^2 + 6y - 4,$
$y = 1,\ y = 6$

7) $x = \cos y,\ x = -2\cos y,$
$y = -\pi,\ y = \frac{\pi}{2}$

8) $x = \sin y,\ x = -\sin y,$
$y = -\frac{\pi}{2},\ y = \frac{5\pi}{6}$

9) $y = -\frac{x^2}{2} - 2x + 2,\ y = -\frac{x^2}{2} - 4x - 6,$
$x = -6,\ x = -1$

10) $y = -\frac{x^3}{2} - \frac{x^2}{2} + 2x,\ y = -x$

11) $y = \sin x,\ y = -\sin x,$
$x = -\pi,\ x = \frac{3\pi}{4}$

12) $x = y^2 + 1,\ x = -\frac{y^2}{2} + y - \frac{1}{2},$
$y = 0,\ y = 2$

13) $x = y^2 - 2y - 2,\ x = 1,$
$y = -2,\ y = 3$

14) $x = 2\sec^2 y,\ x = -2\sec^2 y,$
$y = -\frac{\pi}{4},\ y = \frac{\pi}{4}$

Answers to Area Between Curves (ID: 1)

1) $\int_{-1}^{2}\left(-x^2+4-\left(x^2-2x\right)\right)dx+\int_{2}^{3}\left(x^2-2x-\left(-x^2+4\right)\right)dx$

$=\frac{38}{3}\approx 12.667$

2) $\int_{-2}^{1}\left(-\frac{x^2}{2}-2x+4-\left(\frac{x^2}{2}+2x-1\right)\right)dx+\int_{1}^{2}\left(\frac{x^2}{2}+2x-1-\left(-\frac{x^2}{2}-2x+4\right)\right)dx$

$=\frac{64}{3}\approx 21.333$

3) $\int_{-\pi}^{-\frac{\pi}{2}}\left(\cos x+\cos x\right)dx+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\cos x+\cos x\right)dx$

$=6$

4) $\int_{-\frac{\pi}{2}}^{0}\left(-2\sin x-2\sin x\right)dx+\int_{0}^{\frac{3\pi}{4}}\left(2\sin x+2\sin x\right)dx$

$=8+2\sqrt{2}\approx 10.828$

5) $\int_{-2}^{-1}\left(y^2-2y-5-\left(-y^2+2y+1\right)\right)dy+\int_{-1}^{3}\left(-y^2+2y+1-\left(y^2-2y-5\right)\right)dy$

$=26$

6) $\int_{1}^{4}\left(-y^2+6y-4-\left(-y^2+8y-12\right)\right)dy+\int_{4}^{6}\left(-y^2+8y-12-\left(-y^2+6y-4\right)\right)dy$

$=13$

7) $\int_{-\pi}^{-\frac{\pi}{2}}\left(-2\cos y-\cos y\right)dy+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\cos y+2\cos y\right)dy$

$=9$

8) $\int_{-\frac{\pi}{2}}^{0}\left(-\sin y-\sin y\right)dy+\int_{0}^{\frac{5\pi}{6}}\left(\sin y+\sin y\right)dy$

$=4+\sqrt{3}\approx 5.732$

9) $\int_{-6}^{-4}\left(-\frac{x^2}{2}-4x-6-\left(-\frac{x^2}{2}-2x+2\right)\right)dx+\int_{-4}^{-1}\left(-\frac{x^2}{2}-2x+2-\left(-\frac{x^2}{2}-4x-6\right)\right)dx$

$=13$

10) $\int_{-3}^{0}\left(-x-\left(-\frac{x^3}{2}-\frac{x^2}{2}+2x\right)\right)dx+\int_{0}^{2}\left(-\frac{x^3}{2}-\frac{x^2}{2}+2x+x\right)dx$

$=\frac{253}{24}\approx 10.542$

11) $\int_{-\pi}^{0}\left(-\sin x-\sin x\right)dx+\int_{0}^{\frac{3\pi}{4}}\left(\sin x+\sin x\right)dx$

$=6+\sqrt{2}\approx 7.414$

12) $\int_{0}^{2}\left(y^2+1-\left(-\frac{y^2}{2}+y-\frac{1}{2}\right)\right)dy$

$=5$

13) $\int_{-2}^{-1} \left(y^2 - 2y - 3\right) dy +$
$\int_{-1}^{3} \left(1 - \left(y^2 - 2y - 2\right)\right) dy$
$= 13$

14) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(2\sec^2 y + 2\sec^2 y\right) dy$
$= 8$