LIMITS – concept central to calculus; essential to understand what it means for a function to have a limit, and then how to find the limit of a function.

The limit of $f(x)$ as x approaches some value c from the left side (left-hand limit) is written:

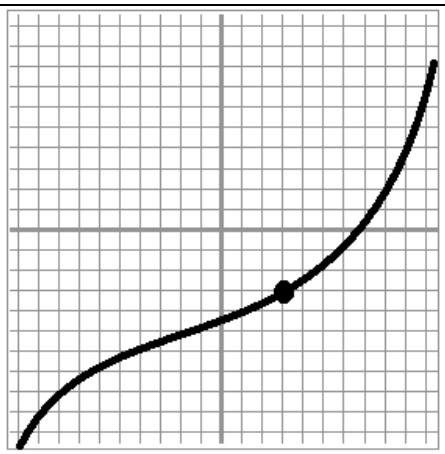
$$\lim_{x \to c^-} f(x)$$

The limit of $f(x)$ as x approaches some value c from the right side (right-hand limit) is written:

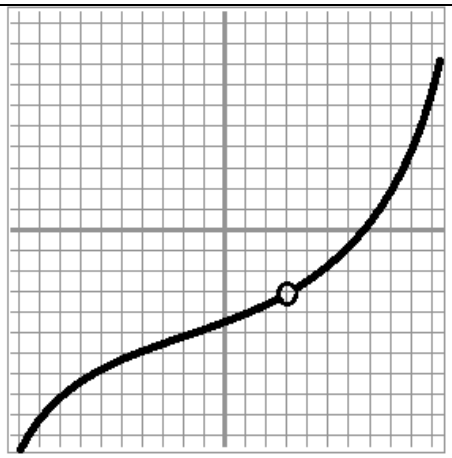
$$\lim_{x \to c^+} f(x)$$

Limit Existence Theorem:

For the function f(x), $\lim_{x \to c} f(x)$ exists if and only if $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$ where c is real.
The limit at a point exists, if the LHL = RHL at that point.
Important! $\lim_{x \to c} f(x)$ does not HAVE to equal $f(c)$.



$\lim_{x \to 3} f(x) = -3$
Why? What's the LHL? RHL?



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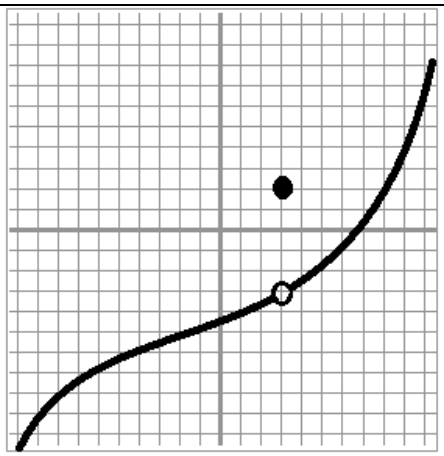

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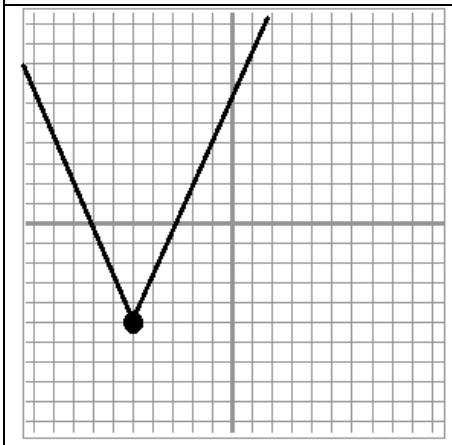
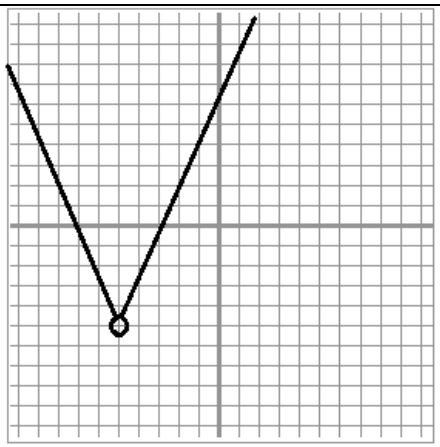


$\lim_{x \to -5} f(x) = -5$
Why? What's the LHL? RHL?



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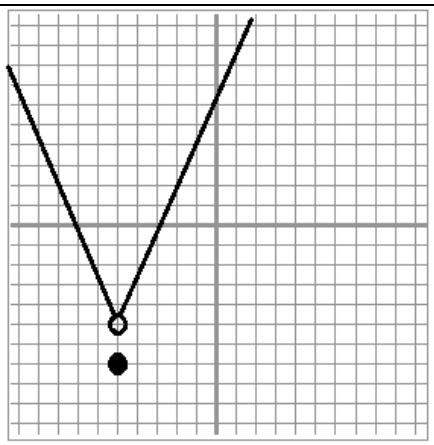

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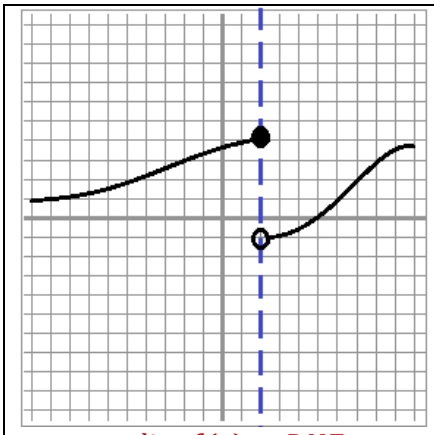
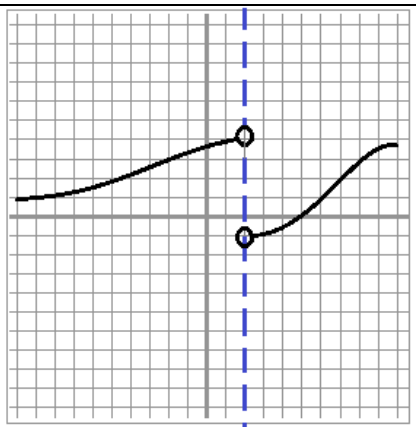


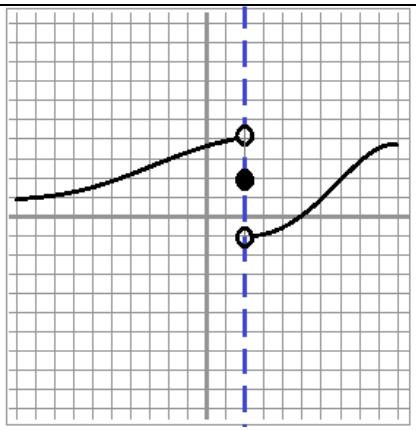
$\lim_{x\to 2} f(x) = DNE$

Why? What's the LHL? RHL?



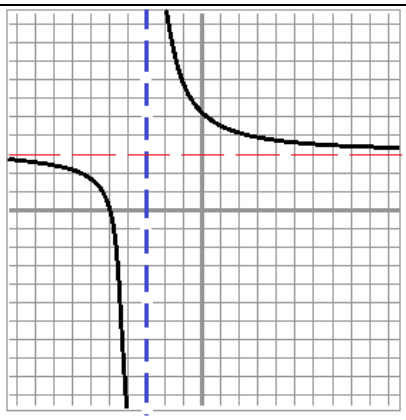
$\lim_{x\to 2} f(x) = DNE$

Why? What's the LHL? RHL?



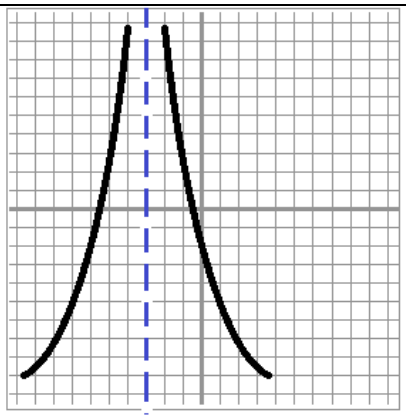
$\lim_{x\to 2} f(x) = DNE$

Why? What's the LHL? RHL?



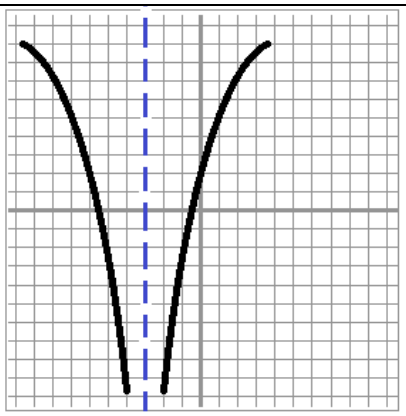
$\lim_{x\to 2} f(x) = 4$

$\lim_{x\to -3} f(x) = DNE$



$\lim_{x\to 2} f(x) = -7$

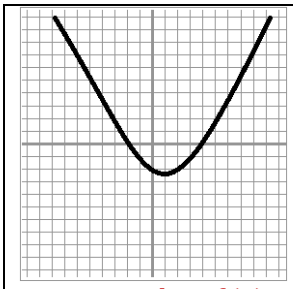
$\lim_{x\to -3} f(x) = \infty$



$\lim_{x\to 2} f(x) = 7$

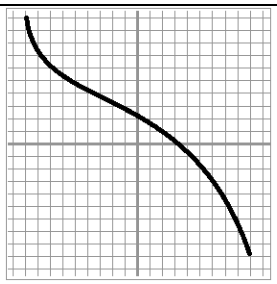
$\lim_{x\to -3} f(x) = -\infty$

End Behavior



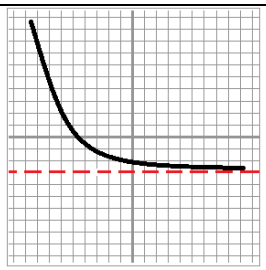
$\lim_{x\to \infty} f(x) = \infty$

$\lim_{x\to -\infty} f(x) = \infty$



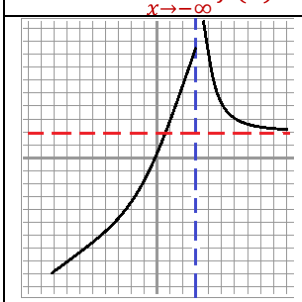
$\lim_{x\to \infty} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = \infty$



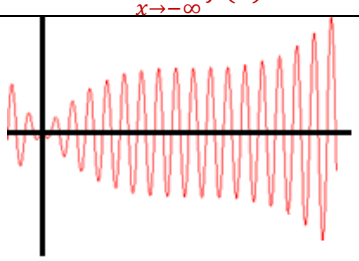
$\lim_{x\to \infty} f(x) = -3$

$\lim_{x\to -\infty} f(x) = \infty$



$\lim_{x\to \infty} f(x) = 2$

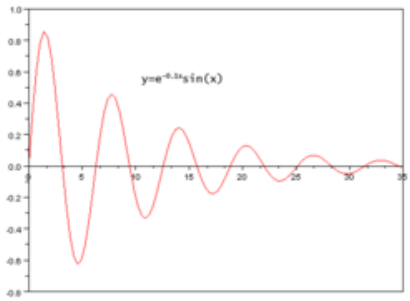
$\lim_{x\to -\infty} f(x) = -\infty$



$\lim_{x\to \infty} f(x) = DNE$

Oscillating Function

Diverging



$\lim_{x\to \infty} f(x) = 0$

Converging